Replacement of PILC/PICAS joints using dynamic programming for optimization and Weibull model for reliability assessment
Hancock, Ian; Zhou, Chengke; Yi, Huajie; Chen, Dong; McDiarmid, Andrew; Eyre-Walker, Ralph
Published in:
2021 IEEE Conference on Electrical Insulation and Dielectric Phenomena (CEIDP)

DOI:
10.1109/CEIDP50766.2021.9705431

Publication date:
2022

Document Version
Author accepted manuscript

Link to publication in ResearchOnline

Citation for published version (Harvard):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
If you believe that this document breaches copyright please view our takedown policy at https://edshare.gcu.ac.uk/id/eprint/5179 for details of how to contact us.
Replacement of PILC/PICAS Joints Using Dynamic Programming for Optimization and Weibull Model for Reliability Assessment

Ian Hancock¹, Chengke Zhou¹, Huajie Yi¹, Dong Chen¹
Andrew Meddiarmid², Ralph Eyre-Walker²

1. School of Computing, Engineering and Built Environment, Glasgow Caledonian University, Cowcaddens Road, Glasgow, G4 0BA, UK
2. Scottish Power Energy Networks (SPEN), 320 St Vincent St, Glasgow, G2 5AD, UK

Abstract: In response to a recent rise in the number of PILC/PICAS joint failures at a utility, the company has set targets to replace a significant number of ageing joints over the next 4 years. This paper aims to provide a methodology for replacement optimization using Dynamic Programming (DP). As part of the methodology, an objective function and constraints is proposed where the goal is to determine the optimal number of yearly replacements which minimizes total cost whilst maintaining an acceptable level of risk. DP is used to find the replacement combination which returns the minimum cost whilst satisfying the constraints. A case study is conducted to investigate the impact cost of replacement, cost of failure and total acceptable replacement limit has on the optimization results. The optimal replacement strategy was obtained. By comparing the optimal replacement strategy against an average unoptimized replacement strategy, it was found that utilities can expect a 14% cost saving and a 40% reduction in the total number of predicted failures by using the replacement optimization methodology detailed in this paper.

I. INTRODUCTION

Since peak cable/joint installations in the UK occurred during the late 50s/60s, many cables/joints are approaching or have exceeded their expected operational life. In recent years, some utility companies have experienced an increasing number of unplanned PILC/PICAS joint failures and according to statistical analysis of the historical failure data for a district of a regional utility company in the UK, a similar trend of increasing PILC/PICAS joint failures can be expected over the next few years as shown in Fig. 1. Although the number of recorded failures for the previous 2 years seems small, the increase in joint failures is consistent across all districts of the utility company representing a much wider problem. To address the issue of rising failures, utilities have set targets to replace a significant number of ageing joints over the next 4 years. However, the problem is deciding which joints should be replaced and how best to prioritize replacement so that risk can be minimized in the most cost effective way.

To address the replacement optimization problem, previous efforts have focused on the application of mathematical optimization techniques such as Exhaustive search, Dynamic Programming and the Greedy algorithm. The Exhaustive search algorithm or ‘Brute Force’ approach solves an optimization problem by iterating for every possible solution to the problem. Whilst this guarantee’s the optimal solution will be found, the Exhaustive search algorithm is inefficient since calculations must be repeated multiple times [1]. Dynamic Programming (DP) also known as dynamic optimization is a powerful method which is considered more efficient than the exhaustive search algorithm [2]. The DP approach can be used to solve a complex problem by breaking it down into smaller sub problems and storing the solutions to those sub problems so they can be re-used later without the need to re-compute. This saves a significant amount of computing power.

Compared to the Greedy optimization algorithm which selects the best option at each stage, the DP algorithm considers the best option at each stage but also considers the solutions to previously solved sub problems in order to compute the optimal solution [3]. In terms of optimality, the greedy algorithm can’t guarantee the optimal solution whereas the DP algorithm is guaranteed to find the optimal solution since it considers every combination of possible decisions and then selects the best combination to satisfy the optimization objective and constraints [4].

The objective of this paper is to provide a methodology for replacement optimization which utilizes dynamic programming to find the optimal replacement schedule which minimizes investment whilst maintaining an acceptable level of risk. In section II, the proposed methodology is outlined and the steps...
required to obtain the optimal replacement decision using DP is explained. In Section III, a case study is provided.

II. METHODOLOGY

The replacement optimization methodology can be summarized by the flowchart in Fig.2. Each stage of the methodology is outlined in the following sub sections. By following the proposed methodology, the optimal replacement decision can be obtained.

A. Weibull Model

The first step of the replacement optimization methodology is to collect data on the cable joints under consideration for replacement. The data should include the age of operational joints and the time-to-failure of joints which have failed. This data is important since it is required to carry out statistical analysis of the joint population. The Weibull distribution is a statistical model for analysis of time-to-failure data and reliability [5]. The Weibull model can ultimately be used to calculate failure probabilities and predict future failures based on the Weibull parameter results. These failure probability and failure prediction calculations are an essential part of the replacement optimization model.

To determine the Weibull shape (β) and scale (η) parameters a Weibull distribution plot must be produced [5]. The X-axis of the plot is the logarithm of time (t). When considering failed joints, time is the time-to-failure which describes the operational period of the joint. For joints that are still in operation, time is the age of the asset from commissioning till the date of study. Time-to-failure data for operational assets is referred to as right censored data. The Y-axis of the Weibull distribution plot is the result of \( \ln(\ln(1/(1-F(t))) \), where \( F(t) \) describes the failure probability. To derive the Weibull parameters, linear fitting of \( \ln(t) \) vs \( \ln(\ln(1/(1-F(t))) \) should take place, where the shape parameter (β) is determined by the slope of the fitted curve and the scale parameter (η) is related to the intercept of the X-axis when \( \ln(\ln(1/(1-F(t))) \) is equal to zero. The linear trend line which relates y and x is given in (1). The Weibull parameters can then be derived from (2) and (3).

\[
y = a + b \cdot x \quad (1)
\]
\[
\beta = b \quad (2)
\]
\[
\eta = \exp(-a/\beta) \quad (3)
\]

Based on the Weibull parameters, failure probability can be determined for each joint using (4) and (5). The predicted number of failures can be obtained using (6) and (7) [6].

\[
F(t+1) = \exp(-((t+1)/\eta)^\beta) \quad (4)
\]
\[
EF_j = F(t+1) - F(t) + (1 - F(t)) \quad (5)
\]
\[
PNoF_j = \sum_{i=1}^{n} EF_j \quad (6)
\]
\[
F(x) = \min(x_{i=1}^{n} CR_i x_i + \sum_{j=1}^{n_i} F(t) CF_i) \quad (7)
\]

Where \( EF_j \) is the estimated failures of the \( j \)th asset, \( PNoF \) is the predicted number of failures, \( j \) is the asset index and \( n \) is the total number of assets.

B. Objective Function and Constraints

The next step of the replacement optimization methodology involves establishing an objective function and identifying constraints. An objective function is a mathematical equation which describes the desired optimization outcome [7]. The aim of the objective function (8) is to minimize the total investment over a period of \( N \) years by finding the optimal combination of yearly replacements required to maintain acceptable failure numbers.

\[
F(x) = \min(x_{i=1}^{N} CR_i x_i + \sum_{j=1}^{n} F(t_j) CF_i) \quad (8)
\]

Where \( i \) is the year index, \( N \) is the number of years under consideration, \( CR_i \) is the cost of replacement in the \( i \)th year, \( x_i \) is the number of replacements in the \( i \)th year, \( j \) is the asset index, \( n_i \) is the number of assets without replacement at the \( i \)th year, \( n_i = n_{i-1} - x_i \), where \( n_0 \) is the total number of assets, \( F(t_j) \) is the failure probability of the \( j \)th asset and \( CF_i \) is the cost of failure in the \( i \)th year.

The objective function is subject to the constraints outlined in (9) ~ (12).

\[
x_i < a \quad (9)
\]
\[
xT = b \quad (10)
\]
\[
PNoF_i \leq YAL_i \quad (11)
\]
\[
PNoF_i \leq TAL \quad (12)
\]

Where \( a \) represents the maximum allowable yearly replacements, \( xT \) is the total number of replacements, \( b \) is the total allowable number of replacements, \( PNoF_i \) is the predicted number of failures in the \( i \)th year, \( PNoF \) is the total predicted number of failures, \( YAL_i \) is the yearly acceptable failure limit and \( TAL \) is the total acceptable failure limit.

C. Dynamic Programing Algorithm

Having established the objective function and constraints, the next stage of the methodology involves finding the acceptable replacement combinations based on the constraints. The DP algorithm starts by considering the maximum allowable replacements for year 1. The algorithm then calculates the predicted number of failures following each replacement...
decision and then checks if the predicted number of failures is less than the yearly acceptable limit. If the predicted number of failures is less than the acceptable limit, the replacement number is stored as an acceptable replacement value for year 1.

In year 2, the algorithm considers all the acceptable replacement values from year 1 and proceeds to find the combination of replacements which result in the predicted number of failures being less than the yearly acceptable limit for year 2. The algorithm repeats this process, storing the acceptable replacement combinations each year for $N$ years.

The DP algorithm takes the acceptable combinations and filters them further by discarding the combinations which fail to meet the total acceptable failure limit and total allowable replacement limit constraints. The DP algorithm then calculates the total cost for the remaining combinations and determines the optimal replacement strategy by finding the replacement combination which minimizes the total cost.

III. CASE STUDY

A. Application of DP Replacement Optimization Model

A numerical example is presented based on data obtained from a regional utility company. The data used in the replacement optimization model is summarized in Table I. The Weibull distribution plot is shown in Fig.3. The Weibull parameters $\beta$ and $\eta$ were found to be 37 and 71.54 respectively.

<table>
<thead>
<tr>
<th>Asset Type</th>
<th>Right Censored</th>
<th>Failures</th>
<th>Total Number (n)</th>
<th>$\beta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PILC/PICAS Joint</td>
<td>402</td>
<td>6</td>
<td>408</td>
<td>37</td>
<td>71.54</td>
</tr>
</tbody>
</table>

![Weibull distribution plot](image)

The cost of replacement and cost of failure has also been obtained from a regional utility company. Expert opinion is that cost of replacement should depreciate around 10% per year. A Breakdown of the cost of replacement and cost of failure for PILC/PICAS joints has been provided in Table II and III [8]. The values selected for the objective function and constraints are listed in Table IV. To solve the replacement optimization problem MATLAB was utilized to find the acceptable replacement combinations and implement the DP algorithm to solve for the optimal replacement strategy.

In this example, four cases are considered for optimization using the DP model. The aim is to analyze the impact variables ($TAL$, ($CR_i$) and ($CF_i$) have on the optimization results. Case 1 is considered the base case and cases 2, 3 and 4 introduce changes to the variables to investigate the impact on optimization results. The results of the DP optimization are given in Table V, where NoR1 ~ NoR4 is the optimal number of replacements for each year, TNoR is the total number of replacements and TNoF is the total number of predicted failures.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$a$</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$b$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$Y_{ALA}$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$Y_{ALB}$</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>$Y_{ALC}$</td>
<td>2.4</td>
<td>2.4</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>$Y_{ALD}$</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>TAL</td>
<td>6.3</td>
<td>6.3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$CR_i$</td>
<td>£8,984</td>
<td>£29,519</td>
<td>£8,984</td>
<td>£29,519</td>
</tr>
<tr>
<td>$CR_2$</td>
<td>£8,086</td>
<td>£26,567</td>
<td>£8,086</td>
<td>£26,567</td>
</tr>
<tr>
<td>$CR_3$</td>
<td>£7,277</td>
<td>£23,910</td>
<td>£7,277</td>
<td>£23,910</td>
</tr>
<tr>
<td>$CR_4$</td>
<td>£6,549</td>
<td>£21,519</td>
<td>£6,549</td>
<td>£21,519</td>
</tr>
<tr>
<td>$CF_i$</td>
<td>£29,519</td>
<td>£8,984</td>
<td>£29,519</td>
<td>£8,984</td>
</tr>
</tbody>
</table>
optimal replacement strategy is to replace 40 assets in years 1 and 2 followed by 20 replacements in year 3 and 0 replacements in year 4.

The reason why case 1 and case 3 have the same result is because the cost of failure is much higher compared to the cost of replacement. The algorithm therefore determined that in both cases the most cost effective option was to replace the maximum allowable number of joints as early as possible. This meant that reducing the total acceptable number of failures in case 3 had no impact on the final optimization result.

To understand the effect cost of failure and cost of replacement has on the optimization results, case 1 and case 2 can be compared. In case 2, the cost of replacement and cost of failure are switched. This represents a scenario when the cost of replacement is higher than the cost of failure. The minimum cost for case 2 was found to be significantly higher than case 1. The optimal replacement combination for case 2 only required 98 replacements compared to 100 replacements in case 1. This suggests that when the cost of replacement is higher than the cost of failure, the algorithm will suggest the minimum number of replacements required to satisfy the constraints.

By comparing case 2 and case 4, the effect of reducing the total acceptable number of failures when the cost of replacement is greater than the cost of failure can be analyzed. Comparing case 2 and case 4 it can be seen that case 2 has a smaller minimum cost however the optimal replacement strategy only requires 98 replacements compared to 100 replacements in case 4. The reason case 4 requires all 100 replacements is because the total acceptable limit was reduced from 6.3 to 6. This explains why the minimum cost for case 4 is slightly higher than case 2 since two more replacements were required to satisfy the total acceptable failure limit constraint.

B. Discussion

To evaluate the benefit of the replacement optimization methodology, the optimal replacement strategy with the minimum cost is compared against an average unoptimized replacement strategy as shown in Table VI. For the average replacement strategy, 25 replacements are made each year.

<table>
<thead>
<tr>
<th>Results</th>
<th>Optimal Strategy</th>
<th>Average Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>NoR1</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>NoR2</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>NoR3</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>NoR4</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>TNoF</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>TNoR</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

By comparing the optimal replacement strategy and the average replacement strategy, it’s clear that a cost saving of £200,980 can be made over 4 years by adopting the replacement optimization methodology outlined in this paper. This equates to a 14% cost saving for utility companies. Furthermore, the optimal replacement strategy reduces the total number of predicted failures by 40%.

IV. Conclusions and Future Work

This paper introduced a replacement optimization methodology which utilized DP to find the optimal replacement strategy for PILC/PICAS joints which minimized investment whilst maintaining an acceptable level of risk. By applying the replacement optimization methodology outlined in this paper, utility companies can expect to make cost savings of 14% and reduce total predicted failures by 40%. The replacement optimization model can be easily tailored to suit the needs of different utility companies by changing the objective function and constraint variables. Given that the predicted number of failures is based on historical failure records, then access to more robust historical failure data will help to improve failure predictions and therefore improve the replacement optimization model.

Future work will involve utilizing the existing replacement optimization methodology to solve a new objective function where the aim is to minimize the predicted number of failures rather than minimize the investment cost.

Acknowledgment

The authors would like to thank Scottish Power Energy Networks (SPEN) for providing the project funding and data.

References


