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Ryu, Inug; Jang, Hanwool; Kim, Dongshin; Ahn, Kwangwon

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Market Efficiency of US REITs: A Revisit

Abstract

The findings on the efficient market hypothesis in the US real estate investment trust (REIT) sector are mixed, and applied methodologies may not be adequate. This paper investigates the weak-form efficient market hypothesis for US REIT stocks. The variance ratio test indicates that the REIT and general stock markets are not efficient in the weak-form. However, the log price series for REIT stocks violates the random walk theory as a model specification. As an alternative, we applied the quantum harmonic oscillator to provide robust evidence. The quantum harmonic oscillator, including the solution for a random walk as a ground state solution, proved to be a better method for testing the efficient market hypothesis. Contrary to variance ratio test, quantum harmonic oscillator provides results that REIT stocks are more efficient than general stocks, and their market efficiency is close to that of bonds. We argue that large market size and substantial institutional ownership of REIT shares have enhanced market efficiency. The findings suggest that arbitrage in the REIT market cannot be achieved simply by analyzing the historical price trend.

Keywords: REITs; stocks; bonds; random walk; quantum harmonic oscillator

1. Introduction

Real properties are traded in two parallel markets [1, 2]. First, physical properties are traded in local property markets in which transaction costs are high and transaction frequency is low [3, 4]. On the other hand, equity real estate investment trusts (REITs), which also invest in physical real properties, trade on the public stock market in the form of REIT shares. They thereby mitigate the illiquid characteristics of real estate transactions. REIT investors can make real estate investments with lower risk, higher liquidity, and moderate returns [5]. Accordingly, the REIT sector has attracted tremendous interest from investors over the last two decades. Specifically, the market capitalization of US REITs has recorded a remarkable average annual growth rate of 13% over the past 19 years.¹ The fast growth in the size of the REIT sector, and its continued trajectory, have raised the question of whether prices adjust to new information instantaneously and any arbitrage opportunity exists.

The existing research tests market efficiency by examining the seasonality and predictability of REIT returns. Colwell and Park [6] provided evidence for seasonality in equity and mortgage REIT returns for the 1964–1986 period. Liu and Mei [7] reported that the expected excess returns on equity REITs are more predictable than those of both small-cap stocks and bonds. Nelling and Gyourko [8] analyzed monthly returns on equity REITs since 1992 and found evidence of predictability. However, Li and Wang [9] employed a multifactor asset-pricing model and documented contrasting evidence for the predictability of mortgage and equity REIT returns: REIT returns are less predictable than those of general stocks despite the strong integration of REITs within the stock market. Therefore, determining market efficiency by examining the seasonality and predictability of REIT returns is still inconclusive.

Another strand of studies examines the random walk hypothesis using the variance ratio test (VRT). The VRT is widely used to examine the correlation structure of volatility [10]. Seck [11] applied the VRT for equity REITs and general stocks during the 1978–1993 period and provided evidence consistent with the random walk hypothesis in both markets. Jirasakuldech and Knight [12] considered mortgage and hybrid REITs as well as equity REITs in their analysis. Contrary to prior studies, they found that the random walk hypothesis does not hold during their study period. Huang et al. [13] further examined the random walk hypothesis of REITs and reported a significant improvement in market efficiency after 2001. Zhou and Lee [14] investigated time-varying market efficiency and found that the REIT market becomes more efficient over the 1980–2009 period. In sum, the VRT provides mixed evidence in light of the random walk hypothesis.

¹ According to the National Association of Real Estate Investment Trusts (NAREIT) and the European Public Real Estate Association, the market capitalization of US REITs has increased from \$138.72 billion at year-end 2000 to \$1,269.96 billion in September 2019. Accordingly, listed REITs accounted for a 4.74% share of the global stock market capitalization as of the third quarter of 2019.

Overall, research on the market efficiency of REIT stocks shows mixed evidence. In particular, it is questionable whether the VRT is a proper methodology for examining the efficient market hypothesis (EMH). Therefore, this study applies the VRT to examine the market efficiency of US REITs and identifies the issues that arise in applying the VRT to REIT stocks. We then propose a quantum harmonic oscillator (QHO) as an alternative to examine the weak-form market efficiency.² The QHO captures market uncertainty through wave functions, and its stochastic differential equation (SDE) incorporates market forces that pull short-run fluctuations back to long-run equilibrium. Specifically, the results from the VRT suggest that REIT and general stock markets are inefficient, whereas the bond market is efficient. However, the log price series of REITs, general stocks, and bonds do not satisfy the random walk theory as a model specification.³ Therefore, we cannot rely on the VRT. We, thus, apply the QHO as an alternative. In contrast to VRT results, those of the QHO suggest that the REIT market is as efficient as the bond market. We argue that large market size and substantial institutional ownership have increased the market efficiency of the REIT sector.

The remainder of this paper is structured as follows. Section 2 describes the data and introduces the methodology. Section 3 discusses the results, and Section 4 concludes the study.

2. Data and Methodology

2.1 Data

We used the Dow Jones US Real Estate Index (DJUSRE), the US Benchmark 10-year Datastream Government Bond Index (BMUS10Y), and the S&P 500 index (S&P 500) to represent the REIT, bond, and general stock sectors, respectively. The starting date for the REIT market is set to January 2000 because the iShares US Real Estate exchange traded fund (ETF), which follows the DJUSRE, commenced in 2000 [15].⁴ Accordingly, the daily closing prices of the three indexes were collected from January 3, 2000, to October 10, 2019, from the Thomson Reuters Datastream.

² An increasing number of quantum finance models have been used to describe the stochastic dynamics of risk assets [23, 25, 42–45].

³ Ahn et al. [23] reported that the null hypothesis of the Cramér goodness-of-fit test, log return data of S&P 500 comes from the Gaussian distribution, can be rejected in the sampling frequency of 1-day, 1-week, and 1-month. Moreover, they provided evidence that the random walk as a model specification severely understates and overstates the probability density of log returns around 0 and in the moderate positive and negative ranges, respectively.

⁴ Following the inception of the DJUSRE ETF, large institutional investors became actively involved in trading real estate ETFs, thus enhancing the speed of price adjustments. This further increased the liquidity of REITs [46, 47]. Since then, the market capitalization of the REITs has skyrocketed.

Table 1 summarizes the descriptive statistics of the log return series. REITs show a higher mean, standard deviation, and kurtosis than the other two asset classes. This implies that REITs are at a relatively immature stage of development [16]. The higher volatility and kurtosis can be explained by the increased market risk precipitated by the inclusion of REITs in the S&P mainstream indices in 2001 [17]. In particular, REITs have negative skewness like that of stocks, indicating a similar investor risk-aversion level in both markets [18–21].

Table 1. Descriptive statistics.

Index	NOs	Mean	SD	Skewness	Kurtosis
DJUSRE	5159	0.052	4.357	-0.201	23.221
BMUS10Y	5159	0.019	1.167	-0.059	2.827
S&P 500	5159	0.034	2.964	-0.227	8.981

Note: Daily log returns from price p_t are annualized as $x_t = 252.5 \times \ln(p_t/p_{t-1})$.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; *NOs*, number of observations; *S&P 500*, S&P 500 index; and *SD*, standard deviation.

2.2 Variance Ratio Test

Frist, the VRT is applied to examine whether the log price series follows a random walk [10, 22]. The key idea behind the VRT is that the variance of the increments in a random walk linearly increases with regard to the sampling interval. If the natural logarithm of a price series, i.e., $Y_t = \ln p_t$, is a pure random walk, the variance of its q^{th} difference should grow proportionally with the number of differences, q . The variance ratio of Y_t , $VR(q)$, is defined as follows:

$$VR(q) = \frac{\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)}$$

with

$$Y_t = \mu + Y_{t-1} + \varepsilon_t, \quad (1)$$

where ε_t follows *i. i. d.* normal distribution with 0 mean and finite variance. $\hat{\sigma}^2$ corresponds to the maximum-likelihood estimator of σ^2 : $\hat{\sigma}^2(1)$, and $\hat{\sigma}^2(q)$ are the variance of the first and q^{th} differences. According to Lo and MacKinlay [10], the formulas for $\hat{\sigma}^2(1)$ and $\hat{\sigma}^2(q)$ are as follows:

$$\hat{\sigma}^2(1) = \frac{1}{(nq - 1)} \sum_{t=1}^{nq} (Y_t - Y_{t-1} - \hat{\mu})^2$$

and

$$\hat{\sigma}^2(q) = \frac{1}{q(nq - q + 1) \left(1 - \frac{q}{nq}\right)} \sum_{t=q}^{nq} (Y_t - Y_{t-q} - q\hat{\mu})^2$$

with

$$\hat{\mu} = \frac{1}{nq} (Y_{nq} - Y_0),$$

where Y_0 and Y_{nq} are the first and last observations of the log price series. The test is performed under both homoscedastic and heteroscedastic specifications. Under the homoscedasticity specification, the asymptotic variance is expressed as follows:

$$\Phi(q) = \frac{2(2q - 1)(q - 1)}{3q(nq)}.$$

Under the heteroscedasticity setting, the asymptotic variance can be expressed as

$$\Phi^*(q) = \sum_{k=1}^{q-1} \left[\frac{2(q-k)}{q} \right]^2 \hat{\delta}(k)$$

with

$$\hat{\delta}(k) = \frac{\sum_{t=k+1}^{nq} (Y_t - Y_{t-1} - \hat{\mu})^2 (Y_{t-k} - Y_{t-k-1} - \hat{\mu})^2}{\left[\sum_{t=1}^{nq} (Y_t - Y_{t-1} - \hat{\mu})^2 \right]^2}.$$

The Z-statistics under homoscedasticity and heteroscedasticity specifications are denoted $Z(q)$ and $Z^*(q)$ and are expressed as follows:

$$Z(q) = \frac{VR(q) - 1}{\sqrt{\Phi(q)}} \sim N(0,1)$$

and

$$Z^*(q) = \frac{VR(q) - 1}{\sqrt{\Phi^*(q)}} \sim N(0,1).$$

The null hypothesis of each test is that the log price series follow a random walk—equivalently, $VR(q) = 1$. If the null hypothesis is rejected with $VR(q) > 1$, then the increment of a log price series is

positively serially correlated and has long-term memory. In contrast, if the null hypothesis is rejected with $VR(q) < 1$, then the increment of a log price series is negatively serially correlated and exhibits a mean reversion property.

2.3 Quantum Harmonic Oscillator

The QHO method, introduced in the work of Ahn et al. [23], models the evolution of the log return series (hereafter, the return series). First, we set up the following SDE⁵:

$$dx = \mu(x, t)dt + \sigma(x, t)dW_t \quad (2)$$

where x represents asset return, $\mu(x, t)$ denotes drift, $\sigma(x, t)$ stands for volatility, and W_t is a standard Wiener process. Then, the Fokker-Planck (FP) equation is obtained from Equation 2 by introducing the probability density function, $\rho(x, t)$, of random variable x at time t [24] as

$$\frac{\partial}{\partial t}\rho(x, t) = \frac{\partial^2}{\partial x^2}(D(x, t)\rho(x, t)) + \frac{\partial}{\partial x}\left(\rho(x, t)\frac{\partial V(x, t)}{\partial x}\right), \quad (3)$$

where $D(x, t) \equiv \sigma^2(x, t)/2$ is the diffusion coefficient, and $V(x, t)$ is the external potential determining the drift term according to $\mu(x, t) \equiv -\partial V(x, t)/\partial x$.

For constant D and time-independent potential $V(x)$, Equation 3 can be expressed in terms of the FP operator:

$$\frac{\partial}{\partial t}\rho(x, t) = \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial x}\frac{\partial}{\partial x} + D\frac{\partial^2}{\partial x^2}\right)\rho(x, t). \quad (4)$$

This can be remedied by transforming the FP equation, Equation 4, into a Schrödinger equation with a Hermitian Hamiltonian [23]:

$$\phi(x, t) \equiv \frac{\rho(x, t)}{\sqrt{\rho_s(x)}}$$

where $\rho_s(x)$ is the stationary solution of the FP equation. Then, the Schrödinger equation can be expressed as

⁵ A SDE is a mathematical model that is widely used to describe various phenomena exhibiting random behavior in a financial market such as geometric Brownian motion [48], stochastic volatility [49], jump processes [50–52], controlled growth processes [53], and processes evolving according to a size-independent proportional growth rate after an exponentially distributed period of time [54].

$$i\hbar \frac{\partial}{\partial \tau} \phi(x, \tau) = \hat{H} \phi(x, \tau) \equiv \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \right) \phi(x, \tau),$$

where \hbar , τ , m , \hat{H} , and $U(x)$ are the reduced Planck's constant, imaginary time, market capitalization, Hermitian Hamiltonian operator, and effective potential,⁶ respectively.

The solution of the Schrödinger equation takes the following form:

$$\phi(x, \tau) = \sum_{n=0}^{\infty} A_n \phi_n(x) \exp\left(-\frac{i}{\hbar} E_n \tau\right),$$

where $\phi_n(x)$ is the solution of the time-independent Schrödinger equation with eigenenergy E_n and amplitude A_n . Then, the steady state solution of the FP equation is given by Ahn et al. [23] as

$$\rho_s(x) = \sqrt{\frac{m\omega}{\pi\hbar}} \exp\left(-\frac{m\omega}{\hbar} x^2\right),$$

where ω is the angular frequency. Finally, the solution of the FP equation takes the following form: in particular, each time-independent term takes the form of a χ distribution:⁷

$$\rho(x, t) = \sum_{n=0}^{\infty} \frac{A_n}{\sqrt{2^n n!}} \sqrt{\frac{m\omega}{\pi\hbar}} \exp(-E_n t) H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right) \exp\left(-\frac{m\omega}{\hbar} x^2\right), \quad (5)$$

where A_n and H_n are the amplitude of the (normalized) solution and the n^{th} Hermite polynomial, respectively.

The random variable x follows the Gaussian, Rayleigh, and Maxwell–Boltzmann distributions for $n = 0, 1, 2$, respectively. They all describe the displacement of a particle in $(n + 1)$ dimensional Euclidean space. In particular, the log return in a ground state ($n = 0$) exactly follows the Gaussian distribution. Thus, the probability assigned to the ground state P_0 indicates the degree to which the log price process is

⁶ As the Taylor expansion of $U(x)$ around the equilibrium x_0 is defined by $dU/dx|_{x_0} = 0$, we can neglect terms of higher order in $x - x_0$ when deviations from the equilibrium are small. Thus, we express

$$U(x) = U(0) + \frac{1}{2} k x^2$$

with $k \equiv d^2U/dx^2|_0$, where we have taken $x_0 \equiv 0$ without loss of generality. Thereby, $U(x)$ is described by a harmonic potential, and the system pins down a harmonic oscillator [23].

⁷ The QHO method has a closed-form solution: that is, a linear combination of an infinite number of χ distributions [23]. Thus, it departs from the normality of log return distribution although it includes the solution of a random walk process as a solution of ground state (for $n = 0$):

$$\rho_n(x) = H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right) \exp\left(-\frac{m\omega}{\hbar} x^2\right).$$

described by a random walk [23, 25]. Therefore, it can be interpreted that the log price process with a larger P_0 is close to a random walk, supporting the weak-form EMH.

3. Results and Discussion

3.1 Variance Ratio Test

Table 2 documents the results of the VRT for the daily log returns with the sampling interval $q = 2, 4, 8, 16$. The random walk hypothesis is rejected under the assumption of homoskedasticity as well as heteroskedasticity in the REIT market. This finding is in line with that of Jirasakuldech and Knight [12], who documented that the random walk hypothesis does not hold in any of their study periods. In particular, the market efficiency of REITs is more similar to that of stocks than of bonds. This result also supports the findings of Seck [11] that there is similarity and substitutability between REITs and stocks in terms of market efficiency. However, the VRT is valid only when the simple volatility-based specification, Equation 1, holds [10]. Therefore, we further investigate the normality of increments of log price series: log return series.

Table 2. Variance ratio test.

		$q = 2$	$q = 4$	$q = 8$	$q = 16$
DJUSRE	$VR(q)$	0.824	0.725	0.621	0.603
	p -value (type 1)	0.000	0.000	0.000	0.000
	p -value (type 2)	0.000	0.001	0.003	0.038
BMUS10Y	$VR(q)$	0.990	0.950	0.927	0.932
	p -value (type 1)	0.459	0.056	0.075	0.265
	p -value (type 2)	0.536	0.112	0.144	0.362
S&P 500	$VR(q)$	0.926	0.848	0.773	0.734
	p -value (type 1)	0.000	0.000	0.000	0.000
	p -value (type 2)	0.004	0.003	0.005	0.031

Note: The p -value is the significance level of the null hypothesis: $VR(q) - 1 = 0$; “type 1” and “type 2” are cases under the assumption of homoscedasticity and heteroscedasticity as a model specification.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; *S&P 500*, S&P 500 index; and *VR*, variance ratio.

We perform the Kolmogorov–Smirnov and Shapiro–Wilk tests for the daily increment ε_t in Equation 1 and summarize the results in Table 3. The null hypothesis of the Kolmogorov–Smirnov and Shapiro–Wilk tests is that the data follow a normal distribution. Both tests reject the null hypothesis at the 1% significance level. Accordingly, we conclude that the assumption of Gaussian white noise on the daily increment of log prices is too strict and ideal. Moreover, the existence of serial correlation in the return series of financial assets, including REITs, is well documented, implying dependency between the increments of log price series [8, 26–28]. Therefore, the VRT is not a proper method of testing the EMH for our datasets. Thus, we employ the QHO framework to further test the market efficiency of REITs.

Table 3. Normality tests for daily log returns.

	Kolmogorov–Smirnov	Shapiro–Wilk
DJUSRE	0.470***	0.767***
BMUS10Y	0.492***	0.976***
S&P 500	0.479***	0.895***

Note: The test statistics are reported. *** indicates the significance level at 1%.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; and *S&P 500*, S&P 500 index.

3.2 Quantum Harmonic Oscillator

The QHO fits the data better than the random walk model (RWM) does for the three markets. First, the QHO precisely estimates both the center and the tail parts, unlike the RWM, as shown in Figure 1: Table 4 further shows that the QHO has a smaller mean absolute error and mean squared error. Second, Figure 2 shows that the residual of the QHO is closer to Gaussian white noise than is that of the RWM, which exhibits a severe deviation from the normality assumption on the daily increment of log prices, especially in both tail parts. Third, the likelihood ratio test indicates that the two likelihoods differ by significantly more than the sampling error, at least for the DJUSRE and S&P 500,⁸ as shown in Table 5. The QHO maximizes over the entire parameter space, whereas RWM imposes some constraints on finding optimal solutions. Thus, we conclude that the QHO is a better method for estimating the unobserved quantity that RWM fails to capture. This is because the QHO incorporates market uncertainty through the properties of

⁸ In the bond market, the goodness of fit of two competing statistical models, based on the ratio of their likelihoods, is not different between the two. Moreover, BMUS10Y can be seen to follow the weak-form EMH regardless of modeling approach, such as QHO and RWM, during the sample period.

wave functions and further introduces friction, namely, market forces that pull short-run fluctuations back to long-run equilibrium [23, 25]. In particular, the solution of the QHO nests that of RWM, as shown in Equation 5, and thus captures not only Gaussian (for $n = 0$) but also non-Gaussian (for $n \geq 1$) features.

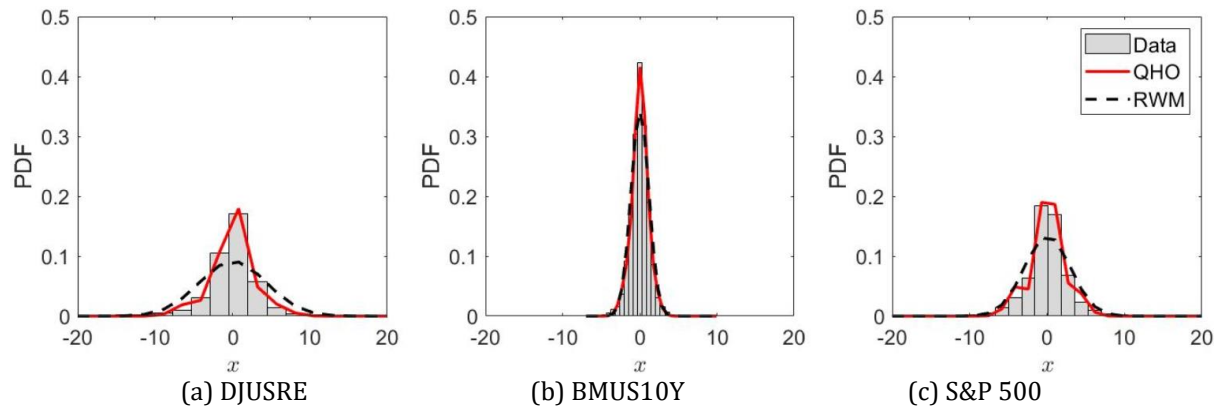


Figure 1. Probability density function of log returns.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; *PDF*, probability density function; and *S&P 500*, S&P 500 index.

Table 4. Model prediction power.

	DJUSRE		BMUS10Y		S&P 500	
	QHO	RWM	QHO	RWM	QHO	RWM
MSE	6.447×10^{-8}	1.739×10^{-6}	1.768×10^{-7}	1.940×10^{-6}	2.883×10^{-7}	1.360×10^{-6}
MAE	1.095×10^{-5}	4.169×10^{-5}	1.953×10^{-5}	4.745×10^{-5}	2.179×10^{-5}	4.091×10^{-5}

Note: MSE and MAE are calculated by

$$MSE = \frac{\sum_{i=1}^M (O_i - F_i)^2}{M} \text{ and } MAE = \frac{\sum_{i=1}^M |O_i - F_i|}{M},$$

where M is the total number of observations, and O_i and F_i are the observed and expected values.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; *MAE*, mean absolute error; *MSE*, mean squared error; *QHO*, quantum harmonic oscillator; *RWM*, random walk model; and *S&P 500*, S&P 500 index.

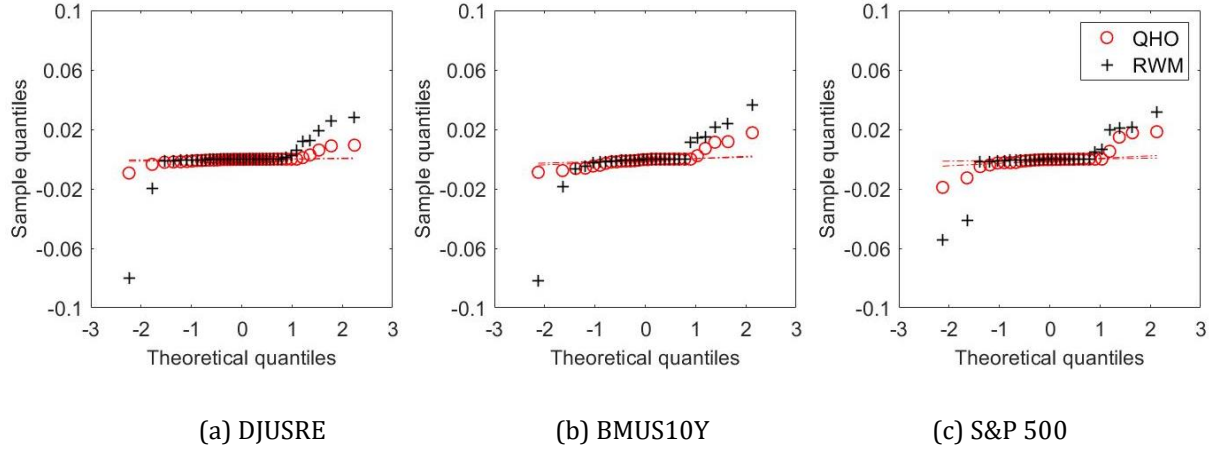


Figure 2. Q–Q plot of the residuals of each model with a regression line.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; and *S&P 500*, S&P 500 index.

Table 5. Likelihood ratio tests for two competing models.

H_0 : There is no difference between the log-likelihood values of QHO and RWM.			
	DJUSRE	BMUS10Y	S&P 500
LR	4351***	-41	2221***

Note: This table reports the results of the log-likelihood ratio test to assess the adequacy of the QHO model over the RWM. The test statistic for LR is defined as

$$LR = -2(\log L_r - \log L_u)$$

where L_u and L_r denote the likelihood of the QHO and RWM, respectively. The unrestricted and restricted log-likelihoods, $\log L_u$ and $\log L_r$, are the values estimated from the two models with their parameters using the maximum-likelihood estimate. The test statistic follows the χ^2 distribution with a degree of freedom 4. *** indicates significance at the 1% level.

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; *LR*, likelihood ratio; and *S&P 500*, S&P 500 index.

Table 6 shows the probability assigned to the ground state P_0 for each asset. P_0 is 98.3%, 99.0%, and 94.4% for REITs, bonds, and stocks, respectively. The P_0 for REITs is 3.9% higher than for stocks and 0.7% lower than for bonds. This suggests that the log return distribution of REITs is much closer to Gaussian than that of stocks and exhibits market efficiency similar to that of bonds. In terms of market capitalization m , the DJUSRE represents only about 1/20th of the value of the S&P 500.⁹ This indicates that angular frequency ω , a proxy for information discovery [25, 29], in the REIT market is much faster

⁹ As of the end of 2018, the S&P 500 market capitalization was \$21.03 trillion, and that of the DJUSRE was \$1.05 trillion [55].

than in the stock market (approximately 10 times more). In contrast to the VRT, our results suggest that the log returns of REITs are well explained by Gaussian distribution, and accordingly, the efficiency of the REIT market is strongly supported.

Table 6. Probability assigned to ground state and angular frequency.

DJUSRE		BMUS10Y		S&P 500	
P_0	$m\omega$	P_0	$m\omega$	P_0	$m\omega$
0.983	9.476	0.990	60.278	0.944	18.466

Abbreviations: *BMUS10Y*, US Benchmark 10-year Datastream Government Bond Index; *DJUSRE*, Dow Jones US Real Estate Index; and *S&P 500*, S&P 500 index.

3.3 Discussion

Our results support the efficiency of the REIT market, at least in the weak-form. This can be explained by a dramatic change in the sector. Following inclusion of REITs in the S&P 500 in January 2001, investors became highly aware of REITs as an asset class in the stock market [13, 30]. Specifically, Figure 3(a) shows that institutional investors have been actively taking ownership stakes in the REIT market over the past two decades [31, 32]. Accordingly, institutional investors have engaged in intensive monitoring of REITs and mitigation of incentives for specialists to misreport. In particular, commercial banks, bond underwriters and investors, and equity investors all monitor REIT performance, which in turn improves the overall information environment by reducing information asymmetry in the market [33–37]. With increased market attention, the REIT sector has achieved high trading volume and market capitalization, as shown in Figure 3(b). Growth of the REIT sector has further improved the communication speed of new market information, thus greater information efficiency has been achieved than in the past, when the sector could be characterized as small and thinly traded [13, 14]. In sum, high-level institutional holdings and a market size that is adequately enough large has improved the market efficiency of REITs.¹⁰

¹⁰ Institutional ownership and trading volume decreased slightly during and 1 year after the global financial crisis; however, trading volume has steadily increased and soared in recent years.

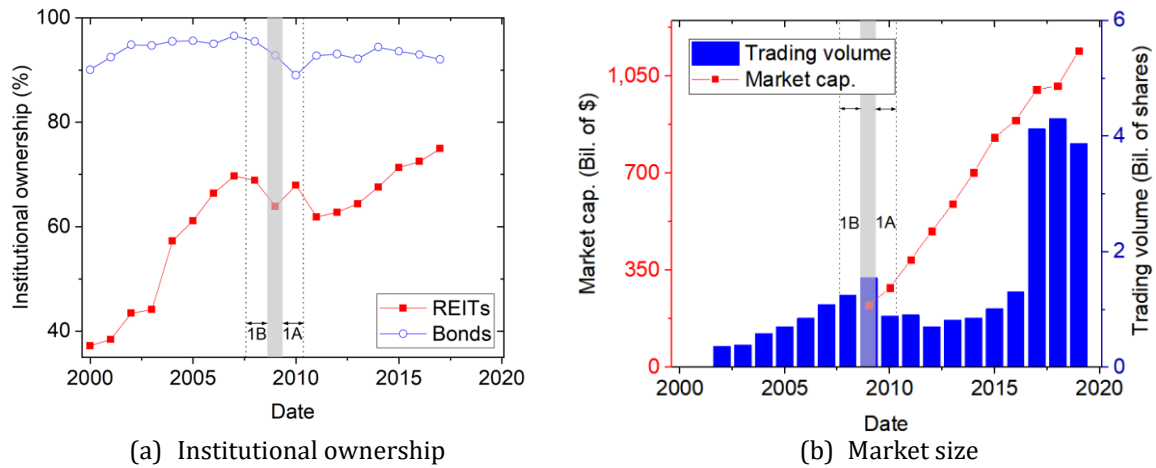


Figure 3. Market development.

Note: Institutional ownership data are from the US Department of the Treasury [38] and Shen [32]. Daily average market capitalization and trading volume are obtained from Thomson Reuters Datastream and Bloomberg, respectively. Due to data limitations, Fig. 3(b) is plotted with different beginning dates. The shaded area is the global financial crisis (GFC) period—September 2008 to May 2009 [20, 39–41]. The periods 1 year before (1B) and after (1A) the GFC are September 3, 2007, to August 29, 2008, and June 1, 2009, to May 31, 2010, respectively.

4. Conclusion

This study examines the market efficiency of the REIT sector compared with that of other financial assets—that is, general stocks and bonds. Our results support the findings of recent studies that show improvement efficiency in the REIT market. The VRT indicates that the REIT and general stock markets appear to be similarly inefficient, whereas the bond market is efficient. However, the dataset does not satisfy the assumption of the VRT: the normality of the increments of the log price series. Therefore, we apply the QHO to find robust evidence of market efficiency—the QHO captures market uncertainty through wave functions and a market force that pulls short-run fluctuations in asset returns back into long-run equilibrium. The QHO, in contrast to the VRT, suggests that the REIT market is more efficient than the general stock market and has achieved efficiency near that of the bond market. We argue that both the larger market size and the greater institutional ownership in the REIT market have enhanced market efficiency. Findings suggest that arbitrage in the REIT sector cannot be achieved by simply analyzing past prices, historical values, and trends, in contrast to the general stock market.

It would be interesting to follow up a QHO approach for screening and identifying a financial market which is at risk. This would allow the regulatory authority to make a timely intervention to get the market back on track. Put differently, the probabilities assigned to higher order eigenstates could be used as early warning indicators because those probabilities measure the distance of the market status from the point at which risky assets trade at their fair value on exchanges. In addition, further research is also encouraged to investigate whether the larger market size and greater institutional ownership in the REIT market have enhanced market efficiency.

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