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Research Article

Maximising the System Spectral Efficiency in a Decentralised 2-Link Wireless Network

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This paper analyses the system spectral efficiency of a 2-link wireless network. The analysis reveals that there exist three operating points that possibly maximise the system spectral efficiency: either both links transmit with maximum power simultaneously or one single link transmits with maximum power while the other is silent. The impact of the chosen multiple access scheme on the system spectral efficiency is also studied: simultaneous transmission or sequential access where the two links share the medium by dedicated time/frequency slots without causing interference. An exhaustive numerical search over a wide range of channel realisations quantifies the gains in system spectral efficiency when choosing either the optimal, single, simultaneous, or sequential medium access. Furthermore, issues regarding the power efficiency are addressed. Finally, the restriction to a 2-link network is relaxed by introducing background interferers, reflecting a multiple link scenario with one dominant interferer. Simulation results indicate that increasing background interference reduces the advantage of sequential over simultaneous transmission.

1. INTRODUCTION

While spectrum is typically regarded as a scarce resource, leading to tremendous efforts to efficiently utilise the dedicated spectrum, measurements indicate that major parts of the spectrum are greatly underutilised [1]. This dilemma, which is attributed to the static and exclusive allocation of dedicated frequency bands to specific systems and/or operators by governmental regulators, has inspired a new research field of dynamic spectrum sharing [2]. However, with various operators sharing the same spectrum, interference mitigation through sophisticated frequency and network planning may no longer be feasible.

One of the key challenges for wireless systems that are decentralised in nature and/or operate in license exempt spectrum is the potential of excessive interference caused by simultaneous transmissions of two (or more) competing radio links [3–5]. In particular, [3] identifies transmit-power control and interference management as one of the three fundamental spectrum-sharing tasks.

The emergence of ubiquitous wireless communication further accelerates the trend towards decentralised and self-organising networks [6–8]. Studies on the capacity of decentralised wireless networks have also addressed the effect of power control. In [9] it is shown that with the constraint of equal transmit powers per node, the network capacity is maximised when nodes transmit with maximum power. In [10], the capacity per node of power constrained ultra-wideband (UWB) network with appropriate power and rate adaptation is shown to increase as the number of nodes increases, under the assumption of large available bandwidth and low transmit powers. In [11], a seemingly contradictory result is presented: the network capacity is maximised when transmitters emit with the minimum transmit power that maintains the network connected. Moreover, the per-user throughput is shown to diminish to zero as the number of users increases. These rather divergent results exemplify that system model assumptions have a profound impact on the obtained results.

In light of the above, the capacity analysis of wireless networks has often been based on asymptotic bounds, idealised assumptions, or implementation of particular transmission schemes. In this paper, we consider a simple case of two simultaneously communicating links, so to derive the optimum power allocation that maximises the sum capacity in a closed form. We demonstrate that there exist three operation modes that possibly maximise the system-spectral efficiency: either both links communicate simultaneously all with maximum power, or one single link transmits with maximum power while the other link is silent. This extends the findings of [9] in the way that *exact* conditions are derived that determine the optimum selection of active links, as a function of the channel characteristics and the maximum available transmit power. To this end, an important observation reported in this paper is that the maximum available transmit power significantly impacts the particular resource allocation strategy that maximises the network capacity. Simulation results, averaged over a wide range of channel realisations, quantify the attainable system spectral efficiency considering the optimum transmission mode that chooses between single and simultaneous transmission. As the optimum selection between single and simultaneous transmission requires full system knowledge about the channel conditions, we also assess the performance when transmitters have partial or no channel knowledge.

As power allocation affects not only the mutual interference to/from competing links, but also the connectivity of the network, resource allocation and link adaptation should be jointly optimised, across the traditional boundaries of system layers [12]. To this end, the problem of accessing one resource unit, where the wireless medium can be either accessed simultaneously or one link is refused access to the channel, is extended to a multiple access scenario, where transmissions may also be scheduled sequentially in mutually orthogonal time-frequency slots. Moreover, issues regarding power-constrained wireless networks are also addressed. One interesting result is that, in case all nodes transmit with the same power, sequential transmission is always more power efficient than simultaneous transmission in terms of system spectral efficiency per Watt in bit/s/Hz/W, irrespective of the channel conditions and the available transmit power.

The restriction to a 2-link network is relaxed in the final part of this work. As a scenario where two links compete for resources in perfect isolation from any other transmission is unlikely to occur in practice, a number of additional links are inserted to produce background interference. By doing so, the findings for the 2-link network are extended to reflect a more realistic multiple link scenario with one dominant interferer.

The remainder of this paper is organised as follows. After introducing the optimisation problem in Section 2, the optimum power allocation that maximises the system spectral efficiency is derived in Section 3. Furthermore, optimal transmission modes in terms of the spectral efficiency per Watt, as well as under a constant total power constraint, are investigated. In Section 4, the distribution of channel realisations for users that are uniformly distributed on a disk is derived. To complement the analysis, simulations are carried out over a wide range of channel realisations in Section 5, including a study of background interferers. Finally, Section 6 draws the conclusions, and relates our work to dynamic spectrum sharing as well as power-constrained networks.

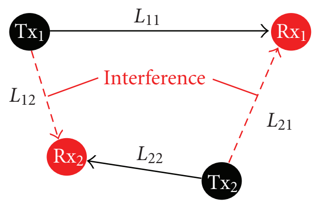


Figure 1: 2-link wireless network. Solid and dashed arrows indicate intended communication links and interference, respectively.

2. PROBLEM STATEMENT

We consider a 2-link communication scenario where nodes Tx_1 and Rx_1 as well as nodes Tx_2 and Rx_2 form a link, as shown in Figure 1. If two links transmit with powers x and y at the same time, their communication is corrupted not only by noise, but also by mutual interference. It is assumed that the two receivers treat interference as additive Gaussian noise. As the utility for the sum capacity, we choose the system spectral efficiency, which is given by

$$C(x, y) = \log_2(1+\gamma_1) + \log_2(1+\gamma_2), \quad (1)$$

where

$$\gamma_1 = \frac{L_{21}}{L_{11}} \cdot \frac{x}{y + NL_{21}}, \qquad \gamma_2 = \frac{L_{12}}{L_{22}} \cdot \frac{y}{x + NL_{12}} \quad (2)$$

denote the signal-to-interference-plus-noise ratio (SINR) at receivers Rx_1 and Rx_2. Moreover, L_{ij} denotes the path loss between transmitter Tx_i and receiver Rx_j, $i, j \in \{1, 2\}$, and N accounts for additive white Gaussian noise (AWGN).

In case the transmit powers of both links are different from zero, $x, y > 0$, the system spectral efficiency $C(x, y)$ in (1) applies for *simultaneous access* where both links are active at the same time. When either $x = 0$ or $y = 0$, one single link is active, referred to as *single transmission*. The corresponding spectral efficiency becomes $C(x, 0)$ or $C(0, y)$.

The objective is to find values of transmit powers, $x, y \in [0, P]$, where P is the maximum available power, for which $C(x, y)$ in (1) is maximised:

$$C_{\max} = \max_{x,y\in[0,P]} C(x, y). \quad (3)$$

Besides power allocation, the scheduling policy and fairness considerations affect the selection of the optimum transmission scheme. With the requirement that both links are granted access to the channel, two multiple access schemes are considered: both links may either access the channel simultaneously, or by *sequential access*. For sequential transmission both links access the channel in dedicated time/frequency slots through TDMA/FDMA, which mitigates interference but halves the available resources. As for sequential transmission the optimum power allocation is to transmit with maximum power, $x = y = P$, its spectral efficiency is given by

$$C_{\text{seq}} = \frac{1}{2}\left[C(P,0) + C(0,P)\right] = \frac{1}{2}\left[\log_2\left(1 + \frac{P}{NL_{11}}\right) + \log_2\left(1 + \frac{P}{NL_{22}}\right)\right]. \tag{4}$$

The total available system powers, for simultaneous transmission on the one hand and single and sequential transmissions on the other, are $2P$ and P, which implies that the comparison in their performance is not fair. To address this issue, a fixed power constraint is introduced, in the way that the power allocation for simultaneous transmission is to be optimised such that the overall system power is constant: $x + y = P$. This translates to the following utility for the selection of the optimum multiple access scheme for equal time-sharing scenario:

$$C_{\text{oma}} = \max_{x \in [\varepsilon, P-\varepsilon]} \left\{C_{\text{seq}}, C(x, P - x)\right\}, \tag{5}$$

where $\varepsilon > 0$ accounts for the minimum transmit power for simultaneous medium access, so that single transmission with $C(0,P)$ or $C(P,0)$ is not allowed. The minimum transmit power ε is introduced to make the comparison (5) meaningful, as $\varepsilon > 0$ ensures that both links are served simultaneously.

3. SYSTEM SPECTRAL EFFICIENCY ANALYSIS

In the following, the power allocation that maximises the system spectral efficiency (3) is derived in Section 3.1. Exact conditions for which simultaneous and single transmissions are preferable are derived in Section 3.2, and constraints for a constant system power are considered in Section 3.3. Finally, the problem of choosing between sequential and simultaneous transmission (5) is addressed in Section 3.4.

3.1. *Optimum power allocation*

In order to determine the power allocation such that $C(x, y)$, $x, y \in [0, P]$ is maximised, it is convenient to cast the sum of two logs in (1) into one single log

$$C(x, y) = \log_2\left[(1 + \gamma_1)(1 + \gamma_2)\right]. \tag{6}$$

Since log is a monotonically increasing function it is sufficient to maximise the argument inside the log of (6). As the first and second derivatives produce an intractable system of nonlinear equations, we attempt to solve (3) by variable transformation. We first show that for a fixed power of one of the transmitters, the other should use either none or full power to maximise $C(x, y)$. Specifically, we show that for a fixed $y = y_0$, $C(x, y_0)$ is maximum only if $x = \{0, P\}$. By variable transformation $z = (x + NL_{12})(y_0 + NL_{21})$, the argument inside the log of (6), $C(x, y_0) = \log_2 g(z)$, can be expressed as

$$g(z) = \frac{A_1 z^2 + B_1 z + D_1}{z}, \tag{7}$$

where

$$\begin{aligned}
A_1 &= \frac{L_{21}}{L_{11}(y_0 + NL_{21})^2},\\
B_1 &= 1 + \frac{(y_0 - NL_{22})L_{21}L_{12}}{(y_0 + NL_{21})L_{11}L_{22}},\\
D_1 &= \frac{L_{12}}{L_{22}} y_0\left(y_0 + NL_{21}\left(1 - \frac{L_{12}}{L_{11}}\right)\right),\\
z &\in \left[NL_{12}(y_0 + NL_{21}),\ (NL_{12} + P)(y_0 + NL_{21})\right].
\end{aligned} \tag{8}$$

In order to find the maxima of $g(z)$, we solve $g'(z) = 0$, to obtain the stationary points $z_{+,-} = \pm\sqrt{D_1/A_1}$. The negative solution, z_-, is not physically valid and is therefore discarded. As $A_1 > 0$, stationary points z_+ only exist for $D_1 > 0$. With $D_1 > 0$ and $z_+ > 0$, the second derivative is positive $g''(z_+) = 2D_1/z^3 > 0$, which means that z_+ is a minimiser. Hence, $g(z)$ is always maximised at boundary values of z. This implies that the maximum of $C(x, y_0)$ in (1), with y_0 fixed, is attained for either $x = 0$ or $x = P$.

Similar reasoning can be applied if we fix $x = x_0$ to show that the argument inside the log of $C(x_0, y)$ is maximised when $y = \{0, P\}$. Therefore, the system spectral efficiency can possibly reach maximum only at the three corner points (x, y): $(0, P)$, $(P, 0)$, and (P, P), as the point $(0, 0)$ is obviously a minimiser. We note that this finding is generally valid for arbitrary channel conditions, transmit, and noise power levels.

3.2. *Choosing between simultaneous and single transmissions*

In order to maximise the system spectral efficiency, nodes must transmit with maximum power. The next step is to derive conditions to choose between simultaneous and single transmissions. The three remaining candidates that maximise (1) are $C(P, 0)$, $C(0, P)$ for single transmission, and $C(P, P)$ for simultaneous transmission.

It is easily shown that $C(P, 0) \geq C(0, P)$ when

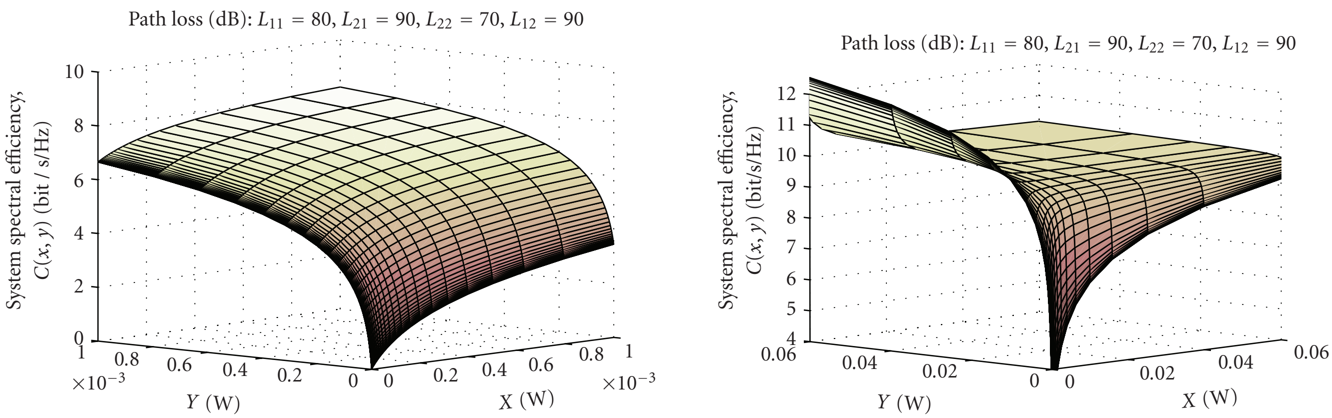
$$L_{11} \leq L_{22}. \tag{9}$$

Furthermore, $C(P, 0) \geq C(P, P)$ when

$$\frac{P}{N} \geq \frac{1}{2L_{22}}\left(L_{12}(L_{11} + L_{21} - L_{22}) + \sqrt{L_{12}^2(L_{22} - L_{11} - L_{21})^2 + 4L_{11}L_{21}L_{22}L_{12}}\right). \tag{10}$$

Likewise, $C(0, P) \geq C(P, P)$ when

$$\frac{P}{N} \geq \frac{1}{2L_{11}}\left(L_{21}(L_{22} + L_{12} - L_{11}) + \sqrt{L_{21}^2(L_{11} - L_{22} - L_{12})^2 + 4L_{11}L_{21}L_{22}L_{12}}\right). \tag{11}$$



(a) (b)

FIGURE 2: System spectral efficiency $C(x, y)$ versus transmit powers $x, y = [0, P]$. (a) Max. power $P = 1$ mW, case where $C(P, P) > C(P, 0)$ and $C(P, P) > C(0, P)$, (b) Max. power $P = 60$ mW, case where $C(P, 0) < C(P, P) < C(0, P)$.

Conditions (10) and (11) indicate that the higher the available transmit power P, the more favourable single transmission becomes. This is intuitively clear by bearing in mind that single transmission is noise limited while simultaneous transmission is interference limited: unlike the SNR, the SINR may not increase when both links increase their power.

Figure 2 plots the system spectral efficiency region $C(x, y)$ over the available power domain $x, y = [0, P]$ for the path loss values $L_{11} = 80$ dB, $L_{21} = 90$ dB, $L_{22} = 70$ dB, and $L_{12} = 90$ dB. Note that the same path loss values are used in both plots in Figure 2. Dependent on the maximum available transmit power P, as well as the channel conditions, the system spectral efficiency is maximised by one of the three corner points, $C(P, P)$, $C(0, P)$, and $C(P, 0)$. As shown in Figure 2(a) the maximum occurs at (P, P) when powers are relatively low, so that both (10) and (11) are not met. In the considered network, the switching points where single is preferred over simultaneous transmission are $P = 0.11$ W for $C(P, 0) \geq C(P, P)$ according to (10), and $P = 9.1$ mW for $C(0, P) \geq C(P, P)$ according to (11). As the available power P increases, the maximum occurs at $(0, P)$, as shown in Figure 2(b). In this case, simultaneous transmission is inferior, due to the lack of interference for single transmission. This illustrates how changing the maximum transmit power influences the choice for the optimal transmission scheme.

Figure 3 shows the plot of the system spectral efficiency region over the available power domain for a different set of path loss values. It is seen that the maximum occurs at (P, P) when interference path losses (L_{12} and L_{21}) are large relative to path losses of the intended links (L_{11} and L_{22}).

As illustrated by Figures 2(a) and 3, in case (P, P) is the optimal operating point, power control (i.e., transmitting with less than maximum power P) does not significantly degrade system spectral efficiency. This observation is important from a practical point of view, especially for power-constrained mobile terminals; although the spectral efficiency is only maximised by transmitting with maximum power, gains tend to be marginal for simultaneous transmission. The reason is that the increase of transmit power on the intended link in turn increases the interference on the other link. This is particularly true in case the system spectral efficiency $C(P, P)$ is dominated by mutual interference.

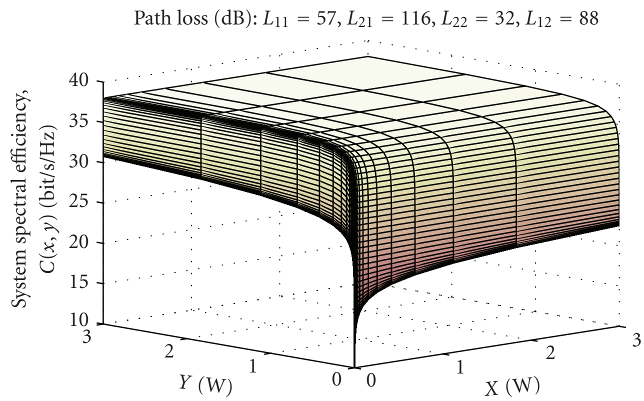


FIGURE 3: System spectral efficiency versus transmit powers where $C(P, P) > C(P, 0)$ and $C(P, P) > C(0, P)$ with max. power $P = 3$ W.

3.3. *Single versus simultaneous transmission under constant system power*

The discussion in Section 3.2 inherently assumes that the simultaneous transmission may consume twice as much power as single transmission. In order to allow for a fair comparison between simultaneous and single transmissions, a constant power constraint is imposed, in the way that the overall transmit power of both transmitters is set to $x + y = P$. To optimise the system spectral efficiency subject to an overall constant power constraint, denoted by $C(x, P - x)$, we show that along the domain line $y = P - x$ there is only one other point other than $(P, 0)$ and $(0, P)$, which is to be checked for optimality.

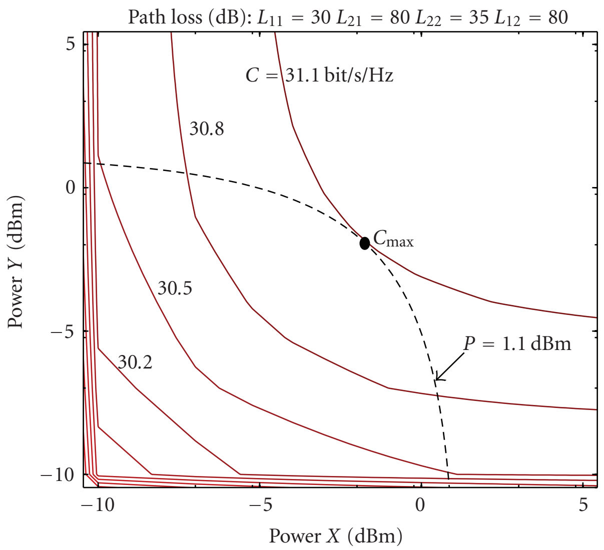


FIGURE 4: Contour plot of the system spectral efficiency over the power domain (x, y). The dashed line depicts the spectral efficiency for constant power $P = 1.1$ dBm.

With $C(x, P-x) = \log_2 h(x)$ and since log is monotonically increasing function, it is sufficient to maximise

$$h(x) = -A_2 + \frac{B_2 x + C_2}{-x^2 + D_2 x + E_2}, \qquad (12)$$

where

$$\begin{aligned}
A_2 &= \left(\frac{L_{21}}{L_{11}} - 1\right)\left(1 - \frac{L_{12}}{L_{22}}\right),\\
B_2 &= \frac{P(L_{21}L_{22} - L_{11}L_{12})}{L_{11}L_{22}}\\
&\quad + \frac{N(L_{21}^2 L_{22} - L_{11}L_{12}^2 - L_{21}^2 L_{12} + L_{21}L_{12}^2)}{L_{11}L_{22}},\\
C_2 &= \frac{L_{12}}{L_{11}L_{22}}(P + NL_{21})\\
&\quad \times (PL_{11} + N(L_{21}L_{22} + L_{11}L_{12} - L_{21}L_{12})),\\
D_2 &= P + N(L_{21} - L_{12}),\\
E_2 &= (P + NL_{21})NL_{12}.
\end{aligned} \qquad (13)$$

Solving $h'(x) = 0$ in order to obtain stationary points provides at most two distinct solutions:

$$x_{1,2} = -\frac{C_2}{B_2} \pm \sqrt{\left(\frac{C_2}{B_2}\right)^2 + \frac{C_2}{B_2}D_2 - E_2}\,. \qquad (14)$$

However, since the valid range for $h(x)$ is $x \in [0, P]$, there is at most one maximiser. This implies that there is at most one point along the line $y = P - x$, other than the end points $x = \{0, P\}$, which maximises $h(x)$.

Figure 4, shows a contour plot of the spectral efficiency versus transmit powers. This plot illustrates how to maximise the spectral efficiency for a given overall power $P = x + y$. The optimal operating point for a particular power line segment $y = P - x$ is the crossing point or tangent to the contour that corresponds to the highest spectral efficiency.

Unfortunately, the possible maximiser of $h(x)$ in (12) has fairly complicated functional dependence on the system parameters. For ease of analysis, we therefore choose the middle point of the line segment, $x = y = P/2$, as an approximation of the optimum power allocation for simultaneous transmission. By observing that $C(x, P - x)$ is often almost constant in the middle part of the diagonal $y = P - x$, as illustrated in Figure 4, this approximation appears justified. With this approximation, there are three transmission modes that need to be checked: $(P, 0)$, $(0, P)$, and $(P/2, P/2)$. The selection between $(P, 0)$ and $(P/2, P/2)$ translates to the following condition:

$$C(P, 0) \geq C\left(\frac{P}{2}, \frac{P}{2}\right). \qquad (15)$$

After some algebraic transformations, condition (15) results in

$$\begin{aligned}
0 \leq\ & \frac{L_{22}P^2}{2N} + NL_{21}L_{12}(L_{22} - L_{11})\\
& + \frac{P}{2}(L_{21}L_{22} + 2L_{22}L_{12} - L_{11}L_{12} - L_{21}L_{12}).
\end{aligned} \qquad (16)$$

Let $P_{1,2}$ denote solutions to the quadratic formula, which are given as

$$\frac{P_{1,2}}{N} = \frac{-a \pm \sqrt{a^2 - 2L_{21}L_{22}L_{12}(L_{22} - L_{11})}}{L_{22}}, \qquad (17)$$

where

$$a = \frac{1}{2}(2L_{22}L_{12} + L_{22}L_{21} - L_{11}L_{12} - L_{21}L_{12})\,. \qquad (18)$$

From the above solution, there are a number of cases that need to be distinguished. Bearing in mind that $P_1 \leq P_2$ in (17), these five cases are the following:

(i) for $a^2 < 2L_{21}L_{22}L_{12}(L_{22} - L_{11})$: condition (15) always holds true;

(ii) for $a^2 \geq 2L_{21}L_{22}L_{12}(L_{22} - L_{11})$,

- (a) $a \geq 0$ and $L_{22} > L_{11}$: both solutions are negative so that (15) is always met, as the transmit power P must always be positive;
- (b) $a \geq 0$ and $L_{22} \leq L_{11}$: only P_2 is non-negative which implies that (15) only holds true for $P \geq P_2$;
- (c) $a < 0$ and $L_{22} > L_{11}$: both solutions of (17) are positive, which implies that (15) is satisfied for $P < P_1$ or $P > P_2$;
- (d) $a < 0$ and $L_{22} \leq L_{11}$: only P_2 is non-negative, so (15) is only met for $P \geq P_2$.

To check for which cases transmission mode $(0, P)$ is superior to $(P/2, P/2)$, the following condition needs to be solved:

$$C(0, P) \geq C\left(\frac{P}{2}, \frac{P}{2}\right) \qquad (19)$$

which holds when the other mode of the single transmission pair is optimal, so we have

$$\frac{P_{1,2}}{N} = \frac{-b \pm \sqrt{b^2 - 2L_{11}L_{21}L_{12}(L_{11} - L_{22})}}{L_{11}}, \tag{20}$$

where

$$b = \frac{1}{2}(2L_{11}L_{21} + L_{11}L_{12} - L_{22}L_{21} - L_{21}L_{12}). \tag{21}$$

Due to symmetry, for (19) the same conditions as for (15) apply, by replacing a in (18) with b in (21). Furthermore, L_{11} is replaced by L_{22}, and L_{21} by L_{12}, and vice versa.

In Figure 3 the spectral efficiency for $C(P/2, P/2)$ exceeds both $C(P, 0)$ and $C(0, P)$. This is confirmed by the condition derived above, by first noting that $L_{11} > L_{22}$ and therefore $C(0, P) > C(P, 0)$. Then from (21), we have $b = -1.25\cdot 10^{20} < 0$ and $b^2 - 2L_{21}L_{22}L_{12}(L_{22}-L_{11}) = 1.57 \cdot 10^{40} > 0$. As $L_{22} < L_{11}$, the solutions P_1 and P_2 are both positive. Thus, the inequality (19) does not hold since $P_1 < P < P_2$ with $P_1 = -30$ dBm, $P_2 = 57$ dBm, and $P = 35$ dBm.

3.4. Sequential versus simultaneous transmission

Having derived the conditions for choosing between single and simultaneous transmissions, we now substitute single by sequential transmission and compare it to simultaneous transmission. Although sequential transmissions is inferior from a system spectral efficiency point of view, since $C_{\text{seq}} \le \max\{C(0, P), C(P, 0)\}$ where C_{seq} is defined in (4), this is nevertheless an interesting case to consider. Unlike single transmission, sequential transmission maintains fairness, as even the user with inferior link quality is served.

Another important aspect is the power efficiency of the network, which is critical for power-constrained mobile terminals, as well as from a regulatory point of view.

3.4.1. Power efficiency of the network

In order to assess the power efficiency, the capacity normalised to the total transmit power, with unit bit/s/Hz per Watt, is introduced. Provided that both nodes transmit with equal power P, we wish to show that, in spectral efficiency per Watt sense, sequential transmission always outperforms simultaneous transmission. In mathematical terms, we wish to show that the following condition always holds:

$$\frac{C_{\text{seq}}}{P} > \frac{C(P, P)}{2P}. \tag{22}$$

After some algebraic manipulation, (22) can be transformed to

$$P^2 + PN(L_{11} + L_{21} + L_{22} + L_{12}) + N^2(L_{11}L_{21} + L_{22}L_{12}) > 0 \tag{23}$$

which always holds since all variables are positive.

We note that on the left-hand side of (22), one link transmits with power P, while on the right-hand side two links are active so that the total power amounts to $2P$. As demonstrated in the following, relaxing the constraint of equal transmit powers per node affects the selection criterion for the optimum multiple access scheme in power-constrained networks.

3.4.2. Sequential versus simultaneous transmission under constant system power

We attempt to identify which multiple access scheme, either sequential or simultaneous transmission, maximises the system spectral efficiency under the constant system power constraint, as formulated in (5). As the optimum power allocation for simultaneous transmission was approximated by $(P/2, P/2)$ in Section 3.3, the utility (5) translates to the condition

$$C_{\text{seq}} \ge C\left(\frac{P}{2}, \frac{P}{2}\right). \tag{24}$$

We note that condition (24) imposes an average transmit power of $P/2$ to individual users, as well as an overall constant power of P to the network. Therefore, condition (24) also applies to the spectral efficiency per Watt, as introduced in Section 3.4.1.

Condition (24) may be transformed to

$$\begin{aligned}\frac{1}{2}\log_2\left(1 + \frac{P}{NL_{11}}\right) + \frac{1}{2}\log_2\left(1 + \frac{P}{NL_{22}}\right) \\ \ge \log_2\left(1 + \frac{L_{21}(P/2)}{L_{11}((P/2) + NL_{21})}\right) \\ + \log_2\left(1 + \frac{L_{12}(P/2)}{L_{22}((P/2) + NL_{12})}\right).\end{aligned} \tag{25}$$

After algebraic manipulation, (25) is expressed as a fifth order polynomial condition:

$$a_5P^5 + a_4P^4 + a_3P^3 + a_2P^2 + a_1P + a_0 \ge 0 \tag{26}$$

where $a_5, a_4, a_3, a_2, a_1, a_0 \in \mathbb{R}$. Therefore, an analytical solution, similar to the conditions presented earlier, is not possible due to the well-known fact that the fifth order polynomials, in general, have no solutions in terms of radicals (this is a consequence of Abel's impossibility theorem [13]). To complicate matters further, each a_k, $k = \{0, \ldots, 5\}$, is a function of the four path losses, L_{ij} with $i, j \in \{1, 2\}$, and additive white Gaussian noise. This simple example shows that even for the apparently simplistic scenario where only two users compete for resources, mathematical analysis may become intractable. We therefore attempt to characterise the selection of the optimum multiple access scheme that approaches C_{oma} in (5) through simulations.

The findings of the system spectral efficiency analysis are briefly summarised in the following.

(i) The optimum power allocation that maximises the system spectral efficiency $C(x, y)$ with $x, y \in [0, P]$ in (1) was derived in Section 3.1. There exist only three operating modes that can possibly maximise (3), these are either both links transmits with maximum power simultaneously (P, P), or one single link transmits with maximum power while the other is silent, $(P, 0)$ or $(0, P)$.

(ii) Exact conditions (9)–(11) are derived in Section 3.2 that identify the transmission mode (either simultaneous or single transmission), as a function of the path losses L_{ij}, $i, j \in \{1, 2\}$, that maximises the system spectral efficiency (3). Generally, higher available transmit powers P tend to favour single transmission.

(iii) A constant system power constraint of P is imposed in Section 3.3. It was shown that there exist also three operating points that possibly maximise $C(x, P - x)$, $x \in [0, P]$: apart from single transmission at $(P, 0)$ and $(0, P)$, there exists at most one other power allocation along the line $x + y = P$ that maximises the system spectral efficiency. As the exact operating point for simultaneous transmission that maximises $C(x, P - x)$, with $x \neq \{0, P\}$, produces unwieldy expressions, a close approximation is obtained by setting $x = y = P/2$.

(iv) To grant both links access to the channel, single transmission is substituted by sequential transmission in Section 3.4. Assuming that the available power *per node* is fixed to P, sequential transmission was shown to be always more power efficient than simultaneous transmission, in spectral efficiency per Watt sense. On the other hand, imposing an constant *overall system power* constraint of P, sequential transmission may not always be superior. Unfortunately, a closed form solution for the optimum multiple access scheme that chooses between sequential and simultaneous transmissions to maximise C_{oma} in (5) does not exist.

4. PATH LOSS DISTRIBUTION FOR USERS UNIFORMLY DISTRIBUTED ON A DISK

The analytical results obtained in the previous section apply to one particular channel realisation in terms of the path losses, L_{ij}, $i, j \in \{1, 2\}$. In order to assess the system level performance of the considered 2-link network, the average system spectral efficiency depends on the chosen location of transmitters and receivers in the network. In the following, the path loss distribution is derived, assuming that users are uniformly distributed within a disk. The disk represents an idealised model for an area with clear-cut boundary such as an airport terminal building or an office space. Specifically, from a set of uniformly distributed users on a disk of radius R, four nodes are randomly selected: two transmitters and two receivers.

In the Appendix the probability density function (pdf) between any two users of distance r on a disk of radius R is derived as follows:

$$f(r) = \frac{4r}{\pi R^2} \arccos\left(\frac{r}{2R}\right) - \frac{2r^2}{\pi R^3}\sqrt{1 - \left(\frac{r}{2R}\right)^2}. \quad (27)$$

Given the distance pdf between two nodes on a disk, $f(r)$, the corresponding path loss distribution (without log-normal shadowing) is derived by variable transformation as described in the following. Distance-dependent path loss is considered, described by

$$l = \alpha + \beta \log_{10}(r) \text{ [dB]}, \quad (28)$$

where l is the path loss in dB, $\beta = 10\eta$ with η being the path loss exponent, r is the distance between the transmitter and receiver, and α is a constant. Then expressing the distance r as a function of l, we obtain

$$r = \rho(l) = 10^{(l-\alpha)/\beta} \in [0, 2R]. \quad (29)$$

The path loss pdf is computed according to the random variable transformation given by

$$f_L(l) = \left|\frac{\mathrm{d}\rho(l)}{\mathrm{d}l}\right| \cdot f(\rho(l)), \quad (30)$$

where the derivative of $\rho(l)$ is

$$\frac{\mathrm{d}\rho(l)}{\mathrm{d}l} = \frac{\ln(10)}{\beta} \cdot 10^{(l-\alpha)/\beta}. \quad (31)$$

Substituting $\rho(l)$ into (27) yields the path loss pdf:

$$f_L(l) = \frac{\ln(10)}{\beta}\frac{4\rho^2(l)}{\pi R^2}\left(\arccos\left(\frac{\rho(l)}{2R}\right) - \frac{\rho(l)}{2R}\sqrt{1 - \frac{\rho^2(l)}{4R^2}}\right), \quad (32)$$

where

$$l \in [\alpha, \alpha + \beta \cdot \log_{10}(2R)]. \quad (33)$$

From (32) it is seen that increasing the disk radius R results in a shift of the path loss pdf $f_L(l)$ to the right.

Theoretical and simulated (with 10^5 iterations) path loss pdfs between two randomly placed nodes on a disk with radius $R = 100$ m, path loss constant $\alpha = 37$ dB and a path loss exponent $\eta = 3$, are plotted in Figure 5.

To make the studies more realistic, log-normal shadowing is added to the path loss model (28). The corresponding path loss pdf is obtained by convolving the pdf of a normal distribution with the pdf (32). To the best of our knowledge, it is not possible to integrate that convolution integral is closed form. The convolution results in the broadening and lowering of the peak of the pdf (32). This is illustrated in Figure 5, where the path loss pdf is plotted, including log-normal shadowing with a standard deviation of 6 dB generated through simulations.

5. PERFORMANCE EVALUATION

5.1. Assessment methodology

In order to supplement the theoretical analysis, the system spectral efficiency of various transmission schemes is elaborated, averaged over the path losses L_{ij}, $i, j \in \{1, 2\}$. The path losses of the two transmitter and receiver pairs are taken

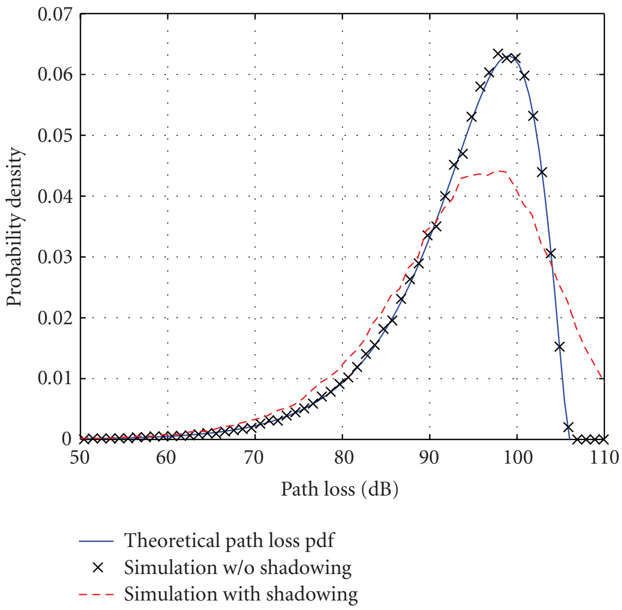


Figure 5: Path loss pdf between two randomly chosen nodes on a disk with radius $R = 100$ m, drawn from a uniform node distribution. The theoretical pdf is shown to agree with the pdf excluding log-normal shadowing obtained via simulation.

from a uniform distribution on a confined circular area, as described in Section 4.

As the analysis showed that the system spectral efficiencies are maximised at the corner points, only the power allocations $(0, P)$, $(P, 0)$, and (P, P) need to be considered. The optimal transmission scheme selects between single and simultaneous transmissions using (9), (10), and (11), such that the maximum system spectral efficiency $C_{\max}$ in (3) is achieved.

Although $C_{\max}$ maximises the system spectral efficiency, perfect system knowledge is required, which involves measurements of all path losses L_{ij}, $i, j \in \{1, 2\}$, and signalling of these locally generated measurements throughout the network. As this involves sophisticated protocols for measurements and signalling, it may not always be feasible to operate the network such that $C_{\max}$ is achieved. Therefore, the expectations of the system spectral efficiencies of simultaneous transmission, $E[C(P, P)]$, and single transmission, $E[\max\{C(P, 0), C(0, P)\}]$, are also evaluated and compared to $E[C_{\max}]$. Unlike simultaneous transmission which does not require any system knowledge, single transmission requires partial channel knowledge to compute $\max\{C(P, 0), C(0, P)\}$. Ensuring that the link with superior spectral efficiency is selected as active link, according to (9), involves measurements and signalling of the pathlosses of the intended links L_{11} and L_{22}.

The expectation of the system spectral efficiency of different multiple access schemes that allow both links to be served is also investigated: sequential transmission, $E[C_{\text{seq}}]$, is compared with simultaneous transmission under a constant system power constraint, $E[C(P/2, P/2)]$. Furthermore, the gap in spectral efficiency to the optimum multiple access scheme $C_{\text{oma}} = \max\{C(P/2, P/2), C_{\text{seq}}\}$ (which corresponds to (5) with $\varepsilon = P/2$) is also elaborated.

While Section 5.2 assumes a 2-link wireless network, this restriction is relaxed in Section 5.3 by considering additional background interferers.

5.2. Simulation for nodes uniformly distributed on a disk

In this section, channel realisations , that resemble uniformly distributed nodes within a disk, are drawn. Figure 5 shows the corresponding path loss pdf between two nodes (32). To evaluate the average system spectral efficiency, Monte Carlo simulations are conducted assuming an AWGN power of $N = -90$ dBm. Distance-dependent path loss (28), with a path loss constant $\alpha = 37$ dB, a path loss exponent of $\eta = 3$, and log-normal shadowing with standard deviation $\sigma = 6$, is assumed.

In Figure 6, the average system spectral efficiency of various transmission schemes are compared for different power levels P and disk radii R. High transmit power levels P generally favour single transmission, while low P favour simultaneous transmission. Furthermore, comparing Figure 6(a) with Figure 6(b), single transmission, $E[\max(C(P, 0), C(0, P))]$, is preferred in small areas (radius $R = 100$ m in Figure 6(a)), and approaches the maximum $E[C_{\max}]$ for high powers P. On the other hand, a larger area (radius $R = 500$ m in Figure 6(b)) is beneficial for simultaneous transmissions and $E[C_{\max}]$ is approached for low powers P. As larger areas imply higher path losses, interference is only significant for higher transmit powers. Hence, the crossing point where single and simultaneous transmissions have the same spectral efficiency is shifted towards a higher power level P. Similar conclusions can be drawn when comparing sequential transmission $E[C_{\text{seq}}]$ with simultaneous transmission under the constant system power constraint $E[C(P/2, P/2)]$: sequential transmission is superior for large powers P and small disk radii R, and approaches the optimum multiple access scheme $E[C_{\text{oma}}] = E[\max\{C(P/2, P/2), C_{\text{seq}}\}]$. The opposite is true for low P and large R, here simultaneous transmission gets close to the optimum, so $E[C(P/2, P/2)] \approx E[C_{\text{oma}}]$.

Table 1 elaborates how the choice of the transmission scheme affects the performance if knowledge about channel conditions (i.e., the path losses between all nodes) is not available. Specifically, the probability that simultaneous transmission achieves a larger system spectral efficiency than single or sequential transmission is determined through simulations. Table 1 indicates that for lower power P, simultaneous transmission tends to be favourable. Likewise, for higher maximum transmit powers P, sequential and single transmissions are superior. Interestingly, sequential transmission, C_{seq}, provides better system spectral efficiency than simultaneous transmission under the constant system power constraint, $C(P/2, P/2)$, even at very low power levels P. This can be explained by the path loss distribution between the transmitter-receiver pairs shown in Figure 5. Due to the skewed shape of the pdf with its distinct peak, path losses are likely to be concentrated around a certain

Table 1: Single and Sequential Transmissions versus Simultaneous Transmission. Disk Radius $R = 100$ m.

Power [dBm]	−30	0	10	20	30
$\Pr(\max\{C(P,0), C(0,P)\} > C(P,P))$ [%]	6	60	81	93	98
$\Pr(C_{seq} > C(P,P))$ [%]	0	22	51	79	94
$\Pr(C_{seq} > C(P/2,P/2))$ [%]	52	54	64	82	95

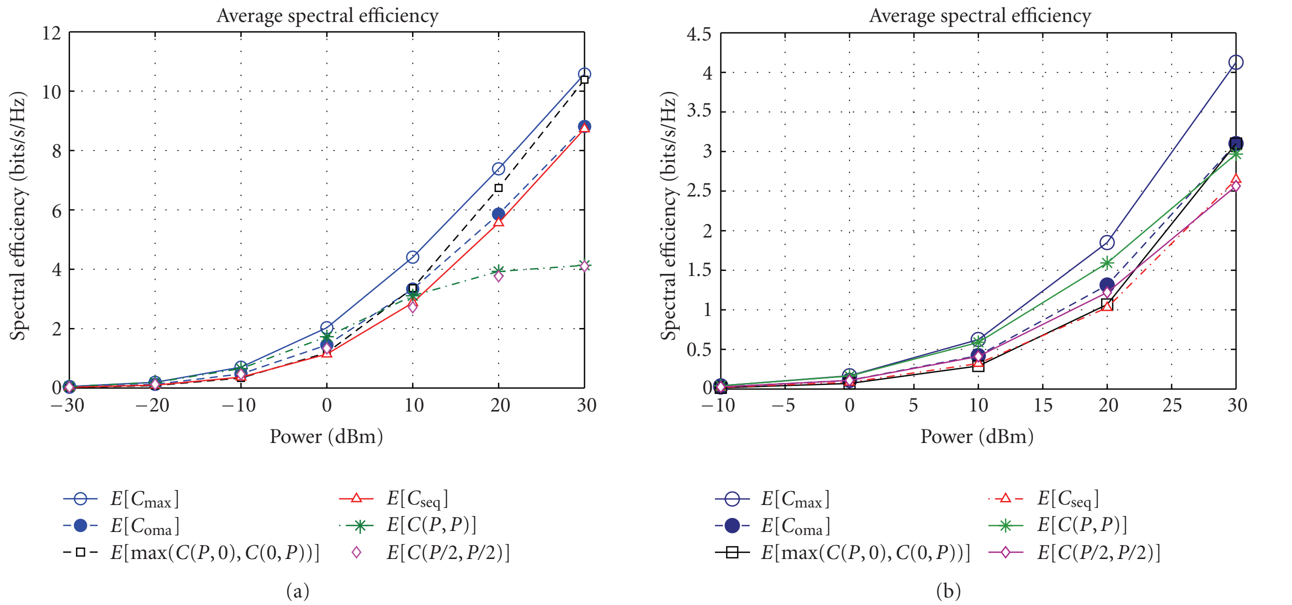


Figure 6: Average system spectral efficiencies for various transmission schemes. Channel realisations are drawn from a uniform distribution of users on a disk with radius R. (a) Disk radius $R = 100$ m, (b) Disk radius $R = 500$ m.

value, which means that intended and interfering links have similar path losses. This gives rise to higher average interference levels, which particularly penalises simultaneous transmission.

While Table 1 indicates the rate of occurrence when a certain transmission scheme is superior, nothing is said about the actual improvement. In order to quantify the attainable gains, we define the normalised increase in system spectral efficiency when simultaneous transmission under the constant system power constraint is preferred over sequential transmission:

$$\mu = \frac{C((P/2),(P/2)) - C_{seq}}{C_{seq}}. \tag{34}$$

Figure 7 shows that the gains provided by simultaneous transmission are rather modest, especially at low transmit powers of $P = -30$ dBm. Here in only about 3% of the cases, the improvement in spectral efficiency exceeds 10% (see point $\mu = 0.1$ from Figure 7). The largest difference is observed for $P = 0$ dBm, even though only for 15% of the points the gains exceed 50%.

In Figure 8 the opposite case is investigated: how much is gained in spectral efficiency if sequential is preferred over simultaneous transmission? The attainable gains are quantified by the normalised increase in overall spectral efficiency when sequential is preferred over simultaneous transmission under the constant system power constraint, defined by

$$\nu = \frac{C_{seq} - C((P/2),(P/2))}{C((P/2),(P/2))}. \tag{35}$$

As shown in Figure 8, in case sequential outperforms simultaneous transmission, it does so significantly. This is because for sequential transmission, there is no interference to disturb the communication of the intended links. For $P = 0$ dBm, over 24% of the points show at least 100% increase in spectral efficiency over simultaneous transmission (see point $\nu = 1$ from Figure 8). Moreover, for larger transmit powers, $P = 30$ dBm, the gains further increase; over 66% of the points exhibit at least 100% increase in spectral efficiency. Finally, at very low power levels of $P = -30$ dBm, the performances of sequential and simultaneous transmissions are rather similar, due to excessive AWGN which dominates the sum capacity (1).

5.3. Including background interference to the 2-link network

The performance evaluations of the considered 2-link network conducted so far inherently favoured sequential

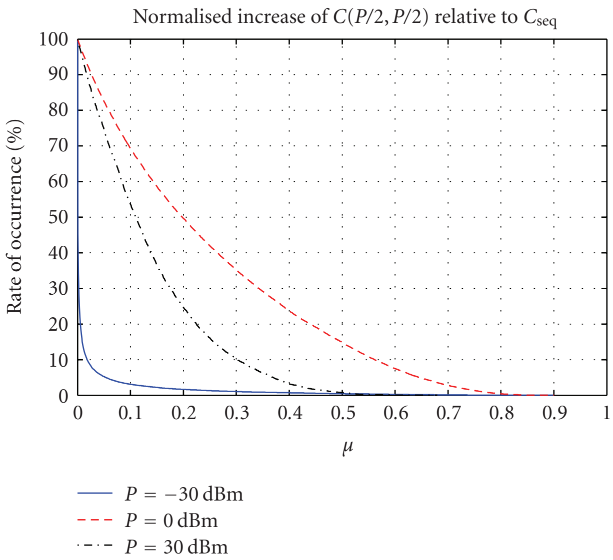


Figure 7: Normalised increase in overall spectral efficiency when simultaneous is preferred over sequential transmission, μ as defined in (34). Disk radius $R = 100$ m.

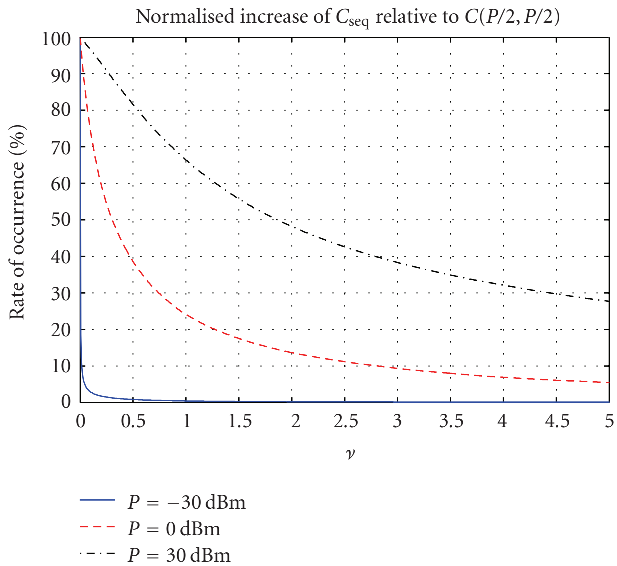


Figure 8: Normalised increase in overall spectral efficiency when sequential is preferred over simultaneous transmission, ν as defined in (35). Disk radius $R = 100$ m.

transmission, since sequential transmission is only limited by noise, while simultaneous access is interference limited. However, a scenario where two links compete for resources in perfect isolation from any other transmission is unlikely to occur in practice. In order to embed the 2-link network into a more realistic setting, a background interferers scenario is introduced with a number of interferers outside a minimum distance R_{ex} away from the receivers Rx_1 and Rx_2, as illustrated in Figure 9. Through the background interferers, the SINRs at the two intended receivers (2) are adjusted as

$$\gamma_1 = \frac{x/L_{11}}{N + (y/L_{21}) + \sum_{j=3}^{N_{int}+2}(P/L_{j1})}, \quad \gamma_2 = \frac{y/L_{22}}{N + (x/L_{12}) + \sum_{j=3}^{N_{int}+2}(P/L_{j2})}, \tag{36}$$

where N_{int} denotes the number of background interferers all of which transmit with power P. For $N_{int} = 0$ the original 2-link network is retained and (36) becomes (2). Furthermore, L_{j1} and L_{j2}, $j = 3, \ldots, N_{int} + 2$, denote the path losses between the background interferers to the two intended receivers. Since the received interference is related to the distance by the path loss (28), an exclusion range R_{ex} around a vulnerable receiver effectively avoids excessive interference of these additional links. The larger R_{ex} the smaller the impact of background interferers, and for $R_{ex} \to \infty$ the 2-link network studied in Section 5.2 is retained. When both intended transmitters Tx_1 and Tx_2 access the channel simultaneously, there will be several interferers, but only one of which is dominant. For sequential transmission, Tx_1 and Tx_2 are orthogonally separated in time and/or frequency, so that both Rx_1 and Rx_2 are only exposed to background interferers. One way of imposing an exclusion region around active receivers is provided by the busy signal concept [14, 15], where receivers broadcast a busy burst in an associated minislot, and each potential transmitter must sense this minislot prior to accessing the channel.

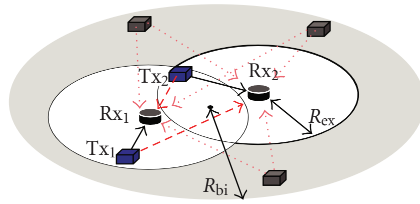
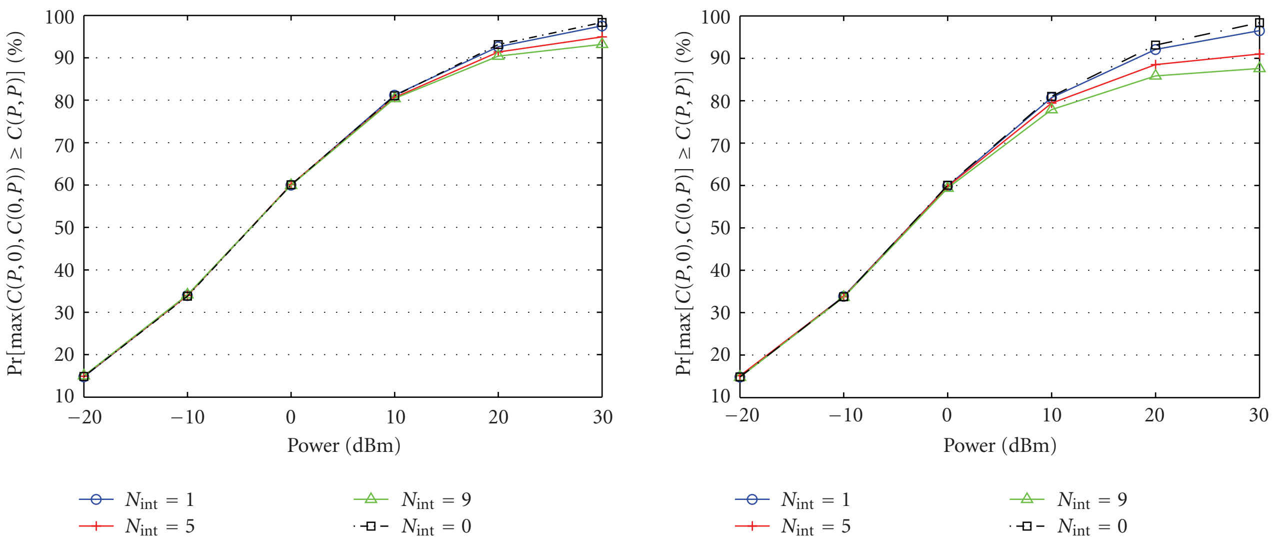


Figure 9: Background interferers scenario: additional transmitters are added with a minimum distance R_{ex} to the intended receivers Rx_1 and Rx_2 . Intended transmitters and receivers are drawn from a disk of radius $R = 100$ m (not shown), while interfering transmitters are drawn from a larger concentric disk of radius $R_{bi} = 1000$ m. Receivers and transmitters are shown as cylinders and rectangular boxes. Solid and dashed arrows account for intended and interfering communication links, respectively.

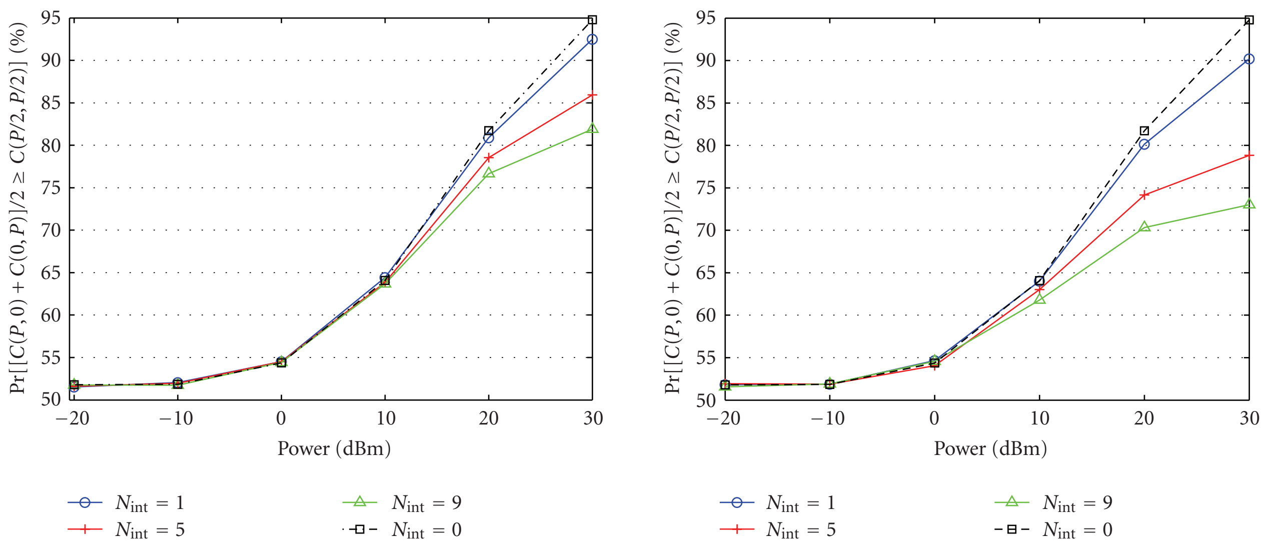
Figures 10 and 11 show results for different number of additional background interferers, power levels P, and exclusion radii R_{ex}. Intended transmitters and receivers are drawn from a disk of radius $R = 100$ m, while interfering transmitters are drawn from a larger concentric disk of radius $R_{bi} = 1000$ m (see Figure 9).

As shown in Figure 10, an increasing number of background interferers modestly degrades the advantage of single transmission at high powers P. By reducing the exclusion range from $R_{ex} = 500$ m in Figure 10(a) to 50 m in Figure 10(b), the impact of background interference somewhat increases. For low powers, on the other hand, there is a diminishing impact of background interference on the choice of the transmission scheme: simultaneous transmission gains over single transmission as P decreases, in analogy to the results of the 2-link network in Table 1.



(a) (b)

FIGURE 10: Percentage of points where single outperforms simultaneous transmission at different power levels and number of background interferers N_{int}. The effect of background interference is only noticeable for high power levels P, and further diminishes by increasing the exclusion region R_{ex}. (a) Exclusion radius $R_{\text{ex}} = 500$ m, (b) Exclusion radius $R_{\text{ex}} = 50$ m.



(a) (b)

FIGURE 11: Percentage of points where sequential outperforms simultaneous transmission at different power levels and different number of background interferers N_{int}. Of note is the significant effect of background interference at high power levels $P > 10$ dBm. (a) Exclusion radius $R_{\text{ex}} = 500$ m, (b) Exclusion radius $R_{\text{ex}} = 50$ m.

In Figure 11, sequential transmission is compared with simultaneous transmission under the constant system power constraint. It is seen that sequential always outperforms simultaneous transmission irrespective of the power level P and the number of background interferers. However, as the number of background interferers increases, the superiority of sequential transmission significantly degrades at $P > 10$ dBm. Moreover, decreasing the exclusion range from $R_{\text{ex}} = 500$ m in Figure 11(a) to 50 m in Figure 11(b) amplifies the effect of additional interferers and therefore compromises the advantage of sequential transmission.

6. SUMMARY AND CONCLUSIONS

In this paper, the system spectral efficiency of a 2-link wireless network is studied by means of mathematical analysis as well as numerical simulations. Although the studied model is simple in nature, a number of fundamental findings are obtained. It is analytically shown that transmitting with maximum power always maximises the system spectral efficiency; either both links transmit simultaneously, or only the link with favourable channel conditions is allowed to transmit. Furthermore, exact conditions, which are functions of path losses, power and noise levels, are established that determine the optimum switching point between single and simultaneous transmissions.

Generally, single transmission compromises fairness. Some level of fairness is introduced by the consideration of sequential transmission, where both links are active on orthogonal resources. Similar to single transmission, sequential transmission tends to outperform simultaneous transmission in an interference limited environment. For example, the average system spectral efficiency of sequential transmission is more than doubled (from 4 bit/s/Hz to 8.8 bit/s/Hz, at transmit power $P = 30$ dBm), when the four nodes are uniformly distributed within a circular area with radius $R = 100$ m. On the other hand, as the node density decreases, simultaneous transmission tends to outperform sequential transmission. Extending the circular region to $R = 500$ m, simultaneous transmission outperforms sequential transmission for all transmit powers considered. However, even in this case the attainable gains of simultaneous over sequential transmission are not significant, especially when the overall transmit power is fixed. When background interferers are considered (up to nine interferers separated by a circular exclusion regions from the two intended receivers), the benefit of sequential transmission somewhat degrades, but remains superior in the majority of cases.

Local knowledge about the interference statistics may be utilised for the choice of the appropriate transmission scheme. As a consequence, power allocation and resource assignment should not be treated independently, giving rise for cross-layer approaches. In this context, it is shown that the higher the maximum transmit power and/or the observed interference, the more advantageous sequential transmission becomes. It is further demonstrated that the available transmit power and the node density impact the interference statistics, and therefore affect the optimum resource allocation strategy. As the available transmit power and/or the node density increase, the SINR tends to be dominated by interference, which typically favours single transmission. These findings are envisaged to provide valuable input for adaptive and interference aware resource allocation algorithms. In the context of dynamic spectrum sharing, it can be concluded that appropriate measures to mitigate interference are not only beneficial to maintain fairness to users with poor channel conditions, but are also meaningful from a spectral efficiency point of view.

Considering the system spectral efficiency per Watt in bit/s/Hz/W, sequential transmission was shown to be always better than simultaneous transmission, regardless of the actual path losses and transmit powers. This particular result suggests that in the case of power-constrained sensor networks, which are mostly characterised by limited available energy and comparably low data rates per node, it may be advantageous to employ resource allocation algorithms that minimise the simultaneous use of radio resources. In case simultaneous transmission is preferred, transmit power levels can often be significantly reduced by hardly sacrificing spectral efficiency.

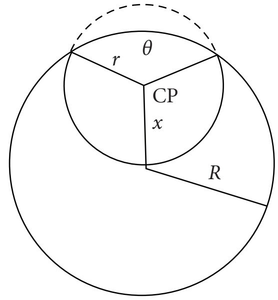


Figure 12: Node CP is shown at distance x from the centre of the disk. Other nodes at distance r are on the ring $r + dr$ centred at CP. The part of the ring outside the disk depicted with the dashed line corresponds to the angle θ.

APPENDIX

The pdf of the distance between two nodes on a disk drawn from a uniform distribution is derived, which is utilised in Section 4. Let point CP be at distance x from the centre of a disk as shown on Figure 12. The probability of having another point at distance r from CP is given by the area of an infinitely thin ring, $r + \mathrm{d}r$ centred at CP, which is $2\pi r\,\mathrm{d}r$, divided by the disk area, πR^2. Considering all the possibilities, the distance distributions are within the range $x \in [0, R]$ and $r \in [0, 2R]$. As r and x vary, parts of the ring might extend outside the confined circular area (the disk), as indicated by the dashed line in Figure 12, which is to be taken into account for the distance distribution. The arc which corresponds to the part of the circle that protrudes outside the area of consideration is expressed as θr, where θ is the angle corresponding to the arc. Then probability density function (pdf) that determines the occurrence of another point at distance r from point CP with distance x from the centre is in the form

$$f(r \mid x) = \begin{cases} \dfrac{2r}{R^2}, & x \leq R - r,\ 0 \leq r \leq R, \\ (2\pi - \theta)\dfrac{r}{\pi R^2}, & x \geq R - r,\ 0 \leq r \leq 2R \end{cases} \tag{A.1}$$

with

$$\theta = 2\ \arccos\left(\frac{R^2 - r^2 - x^2}{2rx}\right). \tag{A.2}$$

To obtain the pdf of the distances between any two points irrespective of the starting position x, the previous equation must be weighed with the probability density of x. This pdf is given as $f(x) = 2x/R^2$ for a uniform distribution of users on a disk, which results in

$$f(r) = \begin{cases} \displaystyle\int_0^{R-r} \frac{4rx}{R^4}\mathrm{d}x + \int_{R-r}^{R} (2\pi - \theta)\frac{2rx}{\pi R^4}\mathrm{d}x, & 0 \le r \le R, \\ \displaystyle\int_{r-R}^{R} (2\pi - \theta)\frac{2rx}{\pi R^4}\mathrm{d}x, & R \le r \le 2R. \end{cases} \tag{A.3}$$

After integration and algebraic manipulations, the following expression is obtained for the distance probability distribution

$$f(r) = \frac{4r}{\pi R^2}\arccos\left(\frac{r}{2R}\right) - \frac{2r^2}{\pi R^3}\sqrt{1 - \left(\frac{r}{2R}\right)^2} \tag{A.4}$$

as the final result in (27).

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