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Robust H_∞ control of an uncertain bilateral teleoperation system using dilated LMIs

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Abstract

A robust state-feedback H_∞ controller is proposed for an uncertain bilateral teleoperation system having norm-bounded parametric uncertainties on mass and damping coefficients of the considered master/slave system. The proposed method ensures robust stability and successful reference tracking and force reflection performance. While the Lyapunov stability is used to ensures asymptotic stability, the H_∞-norm from exogenous input to the controlled output is utilized in satisfying the reference tracking and force reflection. As two performance objectives and robust stability requirement are conflicting with each other, the proposed controller reduces the associated conservatism with dilated linear matrix inequalities (LMIs). Standard and dilated LMI based robust H_∞ state-feedback controllers are performed with a one-degree-of-freedom uncertain master/slave system under reference signal and environmental-induced exogenous force. Numerical simulation results show that the dilated LMI based H_∞-control satisfies lower H_∞-norm than a standard H_∞-control. Moreover, the proposed controller demonstrates a very successful performance in achieving performance objectives despite the stringent norm bounded parameter uncertainties.

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1 Introduction

Teleoperation systems are one of the wide application areas of remote controlled robots that enable us to use human capability in dangerous or inaccessible locations. Teleoperation systems consist of master and slave subsystems, human and environmental-induced exogenous forces, and communication channels. Here, the motion control loop starts at the master side in which an input force is applied by a human operator, and the displacement information of the master is transmitted to the slave through a communication channel. Thus, the slave tries to follow that displacement command. If the slave subsystem is in contact with a remote location (environment), this situation is called "*in-contact motion*". Otherwise, it is called "*free motion*". Also, the force information at the slave can be transmitted back to the master as a force feedback. Hence, the human operator senses the environmental-induced force. This two-way information transmission makes the system bilateral.

The performance objectives of the bilateral teleoperation system consist of reference tracking and force reflection. However, the force reflection conflicts with the stability of the closed-loop system ([Hokayem and Spong(2006)]). Therefore, simultaneous satisfaction of stability and both performance objectives make bilateral teleoperation control a challenging problem. After the first master/slave system developed by [Goertz(1949)], scholars started to work on the control of teleoperation systems. [Sheridan and Ferrell(1963)]'s stability analysis with *move and wait* strategy and addition of the force reflection to this analysis by [Ferrell(1965)] are the first control studies in this area. In later years, many breakthroughs happened in the bilateral teleoperation system control such as Lyapunov based stability analysis ([Miyazaki et al.(1986)]), definition of transparency ([Lawrence(1993)]), and utilization of H_∞ and μ-synthesis techniques ([Leung et al.(1995)]). Over the last decade, the bilateral teleoperation system has been studied with many different advanced control methods. [Du(2013)], designed a state-feedback H_∞ controller and simulated its performance on both free and in-contact motion. [Yang et al.(2015)], used a model predictive control method based on mixed-H_2/H_∞ performance constraints for space teleoperation systems and the proposed controller is compared with other works in the literature.

On the other hand, existence of parameter uncertainties in teleoperation systems is a source of degradation in the performance and even can cause instability. In order to eliminate this problem, many different robust control techniques are offered in the literature. Defining parametric uncertainties as a norm-bounded uncertainty is one of the well accepted methods at the system level. [Xie(1996)], designed a robust output feedback controller for norm bounded parameter uncertain systems. [Parlakçı(2006)], considered norm bounded parameter uncertainties in developing state- and output-feedback controllers and demonstrated the performance of the proposed controller using different uncertain matrices. [Zhang et al.(2012)] used a similar technique to describe uncertain system matrices in LMI based H_∞ control. In [Beikzadeh and Marquez(2018)], the network channel is modelled as norm bounded uncertainties in a nonlinear bilateral teleoperation system.

On the other side, dilated (extended) LMIs play a crucial role in the multi-objective convex optimisation and robust control. Linear matrix inequalities can be dilated by another design variable and the controller can be described with this variable instead of the Lyapunov matrix. Hence, the new variable does not have to be a symmetric matrix and the solution set of LMIs expands. This can make the controller synthesis less conservative, especially in the multi-objective and robust control. [De Oliveira et al.(2002)]'s H_2 and H_∞ controllers for discrete-time systems and [Apkarian et al.(2001)]'s H_2 controller for continuous-time systems are the novel works in dilated LMIs. [Ebihara and Hagiwara(2004)], benefits from dilated LMIs by using different Lyapunov matrices in multi-objective $H_2/\mathbf{D}$-stability controller. Also, [Sajjadi-Kia and Jabbari(2007)] presents a new dilated variation of Bounded Real matrix inequality (MI), invariant set MI and constraint MI. [Pipeleers et al.(2009)] proposed a general methodology for deriving dilated LMIs and provided a straightforward and unified proof technique.

Further to existing methods, this paper considers a robust state-feedback H_∞ controller design problem for teleoperation systems. Lyapunov stability and H_∞ performance constraints are described in terms of dilated LMIs which aim to develop practically applicable and less conservative controller. Also, we address robustness based on the norm bounded type parametric uncertainties. The uncertain parameters are chosen as mass and damping coefficient of the considered master/slave system. Our main motivations for proposing this paper can be summarized as follows:

Robust control of uncertain bilateral teleoperation systems can sometimes be conservative because of the trade-off between performance requirements and robust stability. In contrast to the standard method, the proposed method reduces conservatism significantly. The proposed method relies on H_∞-norm condition. In order to obtain a less conservative solution, a dilated

LMI based formulation is derived for the robust stability and performance of uncertain bilateral teleoperation systems. The benefit of using dilated LMIs, in contrast to the standard LMIs is that, they involves products of an additional unstructured matrix variable and the system's state-space matrices, instead of the symmetric Lyapunov matrix. Therefore, the proposed controller provides less conservative robustness conditions. The proposed dilated H_∞ controller tolerates the same uncertainty bounds at the lower H_∞ norm than the standard H_∞ controller. Moreover, it shows better performance in robust stability, reference tracking and force reflection.

Rest of the paper is organized as follows: Section 2 includes state-space representation of the bilateral teleoperation system having norm-bounded type uncertainties. In Section 3, the standard robust H_∞ controller is presented along with the proposed dilated form. In Section 4, simulation studies are provided to demonstrate the effectiveness of the proposed dilated robust H_∞ controller. Finally, conclusions and future directions are drawn in Section 5.

Notation: A fairly standard notation is used throughout the paper. $\mathbb{R}$ stands for a set of real numbers. $\mathbb{R}^i$ represents i-dimensional vector space. $\mathbb{R}^{n\times m}$ is a set of $n \times m$ dimensional real matrices. $\mathbb{R}^+$ denotes a set of positive real numbers. Q^T denotes the transpose of Q. To simplify notation, $*$ is used to represent a block matrix that is readily inferred by symmetry. $\mathsf{He}\{Q\} = Q + Q^T$. $\mathsf{diag}(\cdot)$ denotes a diagonal matrix. Identity and null matrices are shown as I and 0, respectively. $Q \succ 0 (\succeq, \preceq, \prec 0)$ denotes that Q is a positive definite(positive semidefinite, negative semidefinite, negative definite) matrix.

2 Mathematical modelling of an uncertain bilateral teleoperation system

In this work, we consider a robust H_∞ controller for a bilateral teleoperation system that consists of a one-degree-of-freedom master/slave system, human operator and environmental-induced exogenous forces, as shown in Figure 1. The main performance objectives are chosen as reference tracking and force reflection. In the design process of the robust H_∞ controller, the performance objectives are described in terms of position error between the master and the slave, force error between the human operator and the environment. In

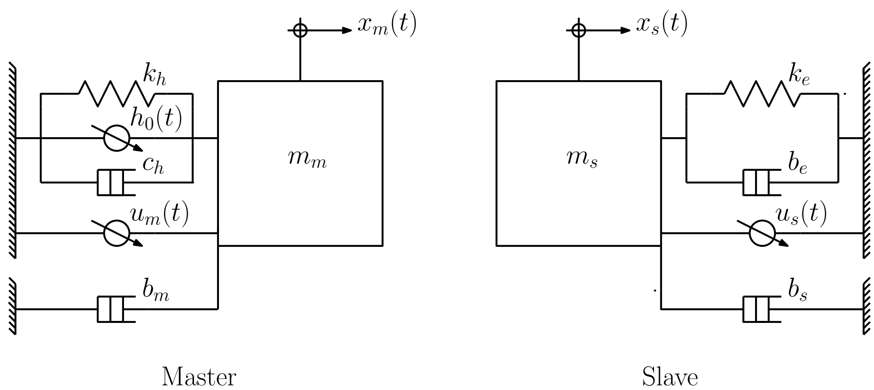

Figure 1: Bilateral teleoperation system

this figure, m_m and m_s are masses of the master and the slave, respectively. $x_m(t)$ is the horizontal displacement of the master, $x_s(t)$ is the horizontal displacement of the slave. b_m stands for the coefficient of the master damping, whereas b_s shows the coefficient of the slave damping. c_h and k_h are coefficients of damping and stiffness, respectively, and describe human hand dynamics. $h_0(t)$ is the reference input force. The human operator exogenous force (reference signal) consists of human hand dynamics and reference input force. b_e and k_e are the coefficient of damping and stiffness, respectively, and describe the environmental-induced exogenous force while it *in-contact*. Finally, $u_m(t)$ and $u_s(t)$ are control forces that are applied to the master and the slave, respectively.

The dynamic equations for the bilateral teleoperation system can be derived from Figure 1 as follows:

$$m_m\ddot{x}_m(t) + b_m\dot{x}_m(t) = u_m(t) + f_h(t), \tag{1}$$
$$m_s\ddot{x}_s(t) + b_s\dot{x}_s(t) = u_s(t) - f_e(t), \tag{2}$$

where

$$f_h(t) = h_0(t) - k_h x_m(t) - c_h\dot{x}_m(t), \tag{3}$$
$$f_e(t) = k_e x_s(t) + b_e\dot{x}_s(t). \tag{4}$$

Then, the state-space form of the bilateral teleoperation system can be obtained from (1)-(2) as follows:

$$\dot{x}_0(t) = A_0x_0(t) + B_0u(t) + H_0w(t) \tag{5}$$

where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b_m}{m_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{b_s}{m_s} \end{bmatrix}, B_0 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_s} \end{bmatrix}, H_0 = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_m} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_s} \end{bmatrix}. \tag{6}$$

Here, $x_0(t) = \begin{bmatrix} x_m(t) & \dot{x}_m(t) & x_s(t) & \dot{x}_s(t) \end{bmatrix}^T$ is the state vector, $w(t) = \begin{bmatrix} f_h(t) & f_e(t) \end{bmatrix}^T$ is the exogenous input vector and $u(t) = \begin{bmatrix} u_m(t) & u_s(t) \end{bmatrix}^T$ is the control input vector.

In order to obtain a satisfactory solution to the reference tracking problem between the reference signal and the master/slave, (5) needs to be extended by introducing extra states of integral of tracking errors as follows:

$$\underbrace{\begin{bmatrix} \dot{x}_0(t) \\ e_{t_m}(t) \\ e_{t_s}(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} A_0 & 0 & 0 \\ -C_{t_m} & 0 & 0 \\ -C_{t_s} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_0(t) \\ \int e_{t_m}(t) \\ \int e_{t_s}(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} B_0 \\ 0 \\ 0 \end{bmatrix}}_{B} u(t) + \underbrace{\begin{bmatrix} H_0 \\ I_t \\ I_t \end{bmatrix}}_{H} w(t) \tag{7}$$

where the tracking error between the reference signal and the master is

$$e_{t_m}(t) = f_h(t) - y_{t_m}(t), \tag{8}$$

where

$$f_h(t) = I_t w(t), \tag{9}$$
$$y_{t_m}(t) = C_{t_m} x_0(t). \tag{10}$$

Similarly, the tracking error between the reference signal and the slave can be written as

$$e_{t_s}(t) = f_h(t) - y_{t_s}(t), \tag{11}$$

where

$$f_h(t) = I_t w(t), \tag{12}$$
$$y_{t_s}(t) = C_{t_s} x_0(t). \tag{13}$$

In equations (10) and (13), $C_{t_m} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$ and $C_{t_s} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ are output matrices which identifies the variable to track $f_h(t)$ and $I_t =$

$\begin{bmatrix}1 & 0\end{bmatrix}$ is an input matrix used to identify the exogenous input reference ([Aktas et al.(2018)]).

In the light of (7), one can consider a state-space representation for the uncertain bilateral teleoperation system as

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + (H + \Delta H(t))w(t), \tag{14}$$

where $x(t) \in \mathbb{R}^n$ is the extended state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^s$ is the exogenous input vector. A, B and H are real, known, time-invariant state-space matrices in appropriate dimensions. $\Delta A(t)$, $\Delta B(t)$ and $\Delta H(t)$ are real, unknown norm bounded state-space matrix functions representing time-varying parameter uncertainties in appropriate dimensions.

Specifically, the uncertainties over the matrices of the system are described as follows:

$$\Delta A(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & p_3\frac{b_m}{m_m} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_4\frac{b_s}{m_s} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{G_a} F(t) \times \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{E_a}, \tag{15}$$

$$\Delta B(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ p_1\frac{1}{m_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p_2\frac{1}{m_s} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{G_b} F(t) \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{E_b}, \tag{16}$$

$$\Delta H(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ p_1\frac{1}{m_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & p_2\frac{1}{m_s} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{G_h} F(t) \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{E_h}, \tag{17}$$

where

$$F(t) = \mathsf{diag}\{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}. \tag{18}$$

Here, p_1, p_2 p_3 and p_4 are maximum parameter uncertainty bounds of $1/m_m$, $1/m_s$, b_m/m_m and b_s/m_s, respectively. G_a, E_a, G_b, E_b, G_h and E_h are real, known, constant uncertainty matrices in appropriate dimensions, which represent the structure of uncertainties, and $F(t)$ is unknown matrix functions with Lebesgue measurable elements $\zeta_1 \in [-1, 1]$, $\zeta_2 \in [-1, 1]$, $\zeta_3 \in [-1, 1]$ and $\zeta_4 \in [-1, 1]$ for all $t \geq 0$.

3 Robust H_∞ controller design using dilated LMIs

Consider a linear time-invariant uncertain bilateral teleoperation system governed by differential equations

$$\mathcal{G}\left\{\begin{array}{lcl} \dot{x}(t) & = & \big(A + \Delta A(t)\big)x(t) + \big(B + \Delta B(t)\big)u(t) + (H + \Delta H(t))w(t) \\ z(t) & = & Cx(t) + Dw(t) \end{array}\right. \tag{19}$$

where $z(t) \in \mathbb{R}^p$ is the collection of controlled outputs, C and D are user defined real, known, time-invariant state-space matrices in appropriate dimensions.

The following Projection Lemma is crucial for the development of dilated LMI based controller.

Lemma 1 *(Projection Lemma:) ([Gahinet and Apkarian(1994)]) Given a symmetric matrix $\mathcal{Z}$ and two matrices $\mathcal{U}$ and $\mathcal{V}$ of appropriate dimensions whose range spaces are independent, there exists an unstructured matrix $\mathcal{X}$ that satisfies*

$$\mathcal{U}^T\mathcal{X}\mathcal{V} + \mathcal{V}^T\mathcal{X}^T\mathcal{U} + \mathcal{Z} \prec 0, \tag{20}$$

if and only if the following projection inequalities with respect to $\mathcal{X}$ are satisfied

$$\mathcal{N}_{\mathcal{U}}^T\mathcal{Z}\mathcal{N}_{\mathcal{U}} \prec 0, \tag{21}$$

$$\mathcal{N}_{\mathcal{V}}^T\mathcal{Z}\mathcal{N}_{\mathcal{V}} \prec 0, \tag{22}$$

where $\mathcal{N}_{\mathcal{U}}$ and $\mathcal{N}_{\mathcal{V}}$ are arbitrary matrices whose columns form a basis of the nullspaces of $\mathcal{U}$ and $\mathcal{V}$, respectively.

Now, consider the following nominal and stable system without control

$$\Sigma \left\{ \begin{array}{rcl} \dot{x} & = & Ax + Hw \\ z & = & Cx + Dw \end{array} \right. \tag{23}$$

Next theorem provides a bounded real lemma by using dilated LMI conditions.

Theorem 1 *(Bounded real-lemma with dilated matrix inequalities) For a given scalar $\gamma_\infty > 0$, Hurwitz system Σ governed by differential equations (23) satisfies $\|\Sigma\|_\infty \prec \gamma_\infty$, if and only if there exist a scalar $\lambda > 0$ and matrices X, $G = G^T \succ 0$ such that*

$$\mathsf{He}\left\{ \begin{bmatrix} -X^T A^T \\ X^T \\ -H^T \\ 0 \end{bmatrix} \begin{bmatrix} \lambda I & I & 0 & 0 \end{bmatrix} \right\} + \begin{bmatrix} 0 & G & 0 & X^T C^T \\ G & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & D^T \\ CX & 0 & D & -I \end{bmatrix} \prec 0. \tag{24}$$

Proof 1 *It is well known that $\|\Sigma\|_\infty \prec \gamma_\infty$ if and only if for all $t \succeq 0$,*

$$\dot{V} + z^T z - \gamma_\infty^2 w^T w \prec 0, \tag{25}$$

is satisfied along the system trajectory (23) ([Boyd et al.(1994)]). Here, $V = x^T W x$ is the Lyapunov function where $W = W^T \succ 0$. In an attempt to derive a dilated version of this condition, let us define an augmented state vector $\phi := \begin{bmatrix} x^T & w^T \end{bmatrix}^T$. Then, in terms of extended state vector, $x = \begin{bmatrix} I & 0 \end{bmatrix} \phi$, $\dot{x} = \begin{bmatrix} A & H \end{bmatrix} \phi$, $w = \begin{bmatrix} 0 & I \end{bmatrix} \phi$ and $z = \begin{bmatrix} C & D \end{bmatrix} \phi$. Based on these descriptions of system variables, one can easily rewrite (25) along the system trajectory (23) as follows:

$$2\phi^T \begin{bmatrix} I \\ 0 \end{bmatrix} W \begin{bmatrix} A & H \end{bmatrix} \phi + \phi^T \begin{bmatrix} C^T \\ D^T \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} \phi + \phi^T \begin{bmatrix} 0 \\ I \end{bmatrix} \left(-\gamma_\infty^2 I\right) \begin{bmatrix} 0 & I \end{bmatrix} \phi < 0 \tag{26}$$

Note that (26) can be further expressed in compact form as follows:

$$\phi^T \begin{bmatrix} I & 0 \\ A & H \\ 0 & I \\ C & D \end{bmatrix}^T \begin{bmatrix} 0 & W & 0 & 0 \\ W & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ A & H \\ 0 & I \\ C & D \end{bmatrix} \phi < 0. \tag{27}$$

For convenience, we express (27) in standard form as,

$$\underbrace{\begin{bmatrix} I & 0 \\ A & H \\ 0 & I \end{bmatrix}^T}_{\mathcal{N}_{\mathcal{U}}^T} \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ C & 0 & D \end{bmatrix}^T \begin{bmatrix} 0 & W & 0 & 0 \\ W & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ C & 0 & D \end{bmatrix}}_{\mathcal{Z}} \underbrace{\begin{bmatrix} I & 0 \\ A & H \\ 0 & I \end{bmatrix}}_{\mathcal{N}_{\mathcal{U}}} \prec 0, \tag{28}$$

where

$$\mathcal{Z} = \begin{bmatrix} C^T C & W & C^T D \\ W & 0 & 0 \\ D^T C & 0 & D^T D - \gamma_\infty^2 I \end{bmatrix}. \tag{29}$$

We will follow Extension-IV introduced by [Pipeleers et al.(2009)], in order to obtain convex conditions which won't introduce extra conservatism. Since the left and right multipliers are identical in (28)*, one can simply choose* $\mathcal{U} = \begin{bmatrix} -A & I & -H \end{bmatrix}$*. Similarly, choosing* $\mathcal{N}_{\mathcal{V}} = \begin{bmatrix} I & -\lambda I & 0 \\ 0 & 0 & I \end{bmatrix}^T$*, leads to the strict matrix inequality* $0 \prec C^T C \prec 2\lambda W$*, the trivial inequality* $0 \prec \gamma_\infty^2 I - D^T D$ *and*

$$C^T C - C^T D E D^T C \prec 2\lambda W, \tag{30}$$

in which we used the definition $E := (D^T D - \gamma_\infty^2 I)^{-1}$*. Note that* $E \prec 0$ *is already implied by the original LMI* (28)*. Defining* $\eta := 0.5\lambda^{-1}$*, the first condition can be rewritten as* $0 \prec \eta C^T C \prec W$*. Hence, choosing* η *sufficiently small, leads to the strict inequality* $W \succ 0$ *which is also imposed by the selection of the Lyapunov function* V*. On the other hand,* $D^T D \prec \gamma_\infty^2 I$ *is implied by the definition of* H_∞ *norm.*

Finally, dividing both sides of (30) *by* 2λ *and using the definition of* η*,* (30) *is equivalent to*

$$\eta(C^T C - C^T D E D^T C) \prec W. \tag{31}$$

Then, applying Schur complement formulae on (31) *gives* (22)*. Note that the left hand side of* (31) *can be less than or equal to zero or strictly greater than zero depending on the matrices* C *and* D*. The first case immediately leads to* $W \succ 0$*. In the latter case, left hand side of* (31) *can be made arbitrarily small by the selecting* η *sufficiently small so that the condition boils down to the strict inequality* $W \succ 0$*. Therefore, the conditions of the dilation do not introduce any conservatism.*

Hence, once again, one can choose $\mathcal{V} = \begin{bmatrix}\lambda I & I & 0\end{bmatrix}$*, and based on the Projection Lemma* (20)*, one can infer that*

$$\mathsf{He}\left\{\begin{bmatrix}-A^T \\ I \\ -H^T\end{bmatrix}\mathcal{X}\begin{bmatrix}\lambda I & I & 0\end{bmatrix}\right\} + \begin{bmatrix}C^TC & W & C^TD \\ W & 0 & 0 \\ D^TC & 0 & D^TD-\gamma_\infty^2 I\end{bmatrix} \prec 0, \tag{32}$$

Then, respectively, defining $X := \mathcal{X}^{-1}$*,* $G := X^TWX$ *and applying a congruence transformation on (32) with a transformation matrix* $\mathsf{diag}\{X, X, I\}$ *gives*

$$\mathsf{He}\left\{\begin{bmatrix}-X^TA^T \\ X^T \\ -H^T\end{bmatrix}\begin{bmatrix}\lambda I & I & 0\end{bmatrix}\right\} + \begin{bmatrix}X^TC^TCX & G & X^TC^TD \\ G & 0 & 0 \\ D^TCX & 0 & D^TD-\gamma_\infty^2 I\end{bmatrix} \prec 0. \tag{33}$$

Finally, applying Schur complement formulae on (33) gives (24). $X^T + X < 0$ *in* (33) *guarantees that* X *is invertible. This concludes the proof.*

Remark 1 *The development of a design procedure without any additional conservatism is crucial for the effectiveness of the dilation. It is well known that* (28) *relies on the positive definiteness of the Lyapunov matrix* ($W \succ 0$) *and the* H_∞*-norm constraints same as the standard method. While* (31) *leads to* $W \succ 0$*, the proposed dilated method does not introduce any extra constraint to the standard* H_∞ *design. Therefore, the proposed dilation method does not introduce any conservatism.*

Replacing A in (24) with $A + BK$ and using the definition $L := KX$ in place, one can obtain an H_∞ state-feedback controller of the form $u = Kx$ for the nominal system

$$\begin{aligned}\dot{x} &= Ax + Bu + Hw \\ z &= Cx + Dw\end{aligned} \tag{34}$$

by using the following Corollary:

Corollary 1 *The closed-loop system*

$$\begin{aligned}\dot{x} &= (A + BK)x + Hw \\ z &= Cx + Dw\end{aligned} \tag{35}$$

is asymptotically stable and has an H_∞-norm from w to z less than $\gamma_\infty > 0$, if and only if there exist a scalar $\lambda > 0$ and matrices X, $G = G^T \succ 0$ and L such that

$$\mathsf{He}\left\{\begin{bmatrix} -X^TA^T - L^TB^T \\ X^T \\ -H^T \\ 0 \end{bmatrix}\begin{bmatrix} \lambda I & I & 0 & 0 \end{bmatrix}\right\} + \begin{bmatrix} 0 & G & 0 & X^TC^T \\ G & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & D^T \\ CX & 0 & D & -I \end{bmatrix} \prec 0. \tag{36}$$

Then, the controller is given by $u = Kx = LX^{-1}x$.

The following Lemma is needed for the development of robust controller in Theorem 2 and Theorem 3.

Lemma 2 *([Xie(1996)]) Given a symmetric matrix $\mathcal{Q}$ and matrices $\mathcal{H}$, $\mathcal{F}$ and $\mathcal{E}$ of appropriate dimensions,*

$$\mathcal{Q} + \mathcal{H}\mathcal{F}\mathcal{E} + \mathcal{E}^T\mathcal{F}^T\mathcal{H}^T \prec 0, \tag{37}$$

for all matrices satistfying $\mathcal{F}^T\mathcal{F} \preceq I$, if and only if there exists some $\epsilon > 0$ such that

$$\mathcal{Q} + \epsilon\mathcal{H}\mathcal{H}^T + \epsilon^{-1}\mathcal{E}^T\mathcal{E} \prec 0. \tag{38}$$

Let us consider the closed-loop uncertain system which is controlled by a state-feedback control $u = Kx$ as follows:

$$\begin{array}{rcl} \dot{x} & = & \big(A + \Delta A(t) + \big(B + \Delta B(t)\big)K\big)x + \big(H + \Delta H(t)\big)w \\ z & = & Cx + Dw \end{array} \tag{39}$$

The following theorem provides a standard robust state-feedback H_∞ controller solution to the uncertain system (14).

Theorem 2 *(Robust H_∞ controller with standard linear matrix inequalities) The closed-loop uncertain system (39) is asymptotically stable and has an H_∞-norm from w to z less than $\gamma_\infty > 0$, if and only if there exist scalars $\epsilon_4 > 0$, $\epsilon_5 > 0$ and $\epsilon_6 > 0$ and matrices S and $Q = Q^T \succ 0$ such that*

$$\begin{bmatrix} \Omega_{11} & H & QC^T & QE_a^T & S^TE_b^T & 0 \\ * & -\gamma_\infty I & D^T & 0 & 0 & E_h^T \\ * & * & -\gamma_\infty I & 0 & 0 & 0 \\ * & * & * & -\epsilon_4 I & 0 & 0 \\ * & * & * & * & -\epsilon_5 I & 0 \\ * & * & * & * & * & -\epsilon_6 I \end{bmatrix} \prec 0 \tag{40}$$

where

$$\Omega_{11} = AQ + QA^T + BS + S^T B^T + \epsilon_4 G_a G_a^T + \epsilon_5 G_b G_b^T + \epsilon_6 G_h G_h^T.$$

Then, the controller is given by $u = Kx = SQ^{-1}x$.

Proof 2 *See in Appendix.*

The following theorem provides a dilated robust state-feedback H_∞ controller solution to the uncertain system (14).

Theorem 3 *(Robust H_∞ controller with dilated linear matrix inequalities) The closed-loop uncertain system (39) is asymptotically stable and has an H_∞-norm from w to z less than $\gamma_\infty > 0$, if and only if there exist scalars $\lambda > 0$, $\epsilon_1 > 0$, $\epsilon_2 > 0$ and $\epsilon_3 > 0$ and matrices X, $G = G^T \succ 0$ and L such that*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & -\lambda H & \Theta_{14} & \Theta_{15} & \Theta_{16} & 0 \\ * & \Theta_{22} & -H & 0 & 0 & 0 & 0 \\ * & * & -\gamma_\infty^2 I & D^T & 0 & 0 & -E_h^T \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\epsilon_2 I & 0 \\ * & * & * & * & * & * & -\epsilon_3 I \end{bmatrix} \prec 0 \qquad (41)$$

where

$$\begin{aligned}
\Theta_{11} &= -\lambda(AX + X^T A^T + BL + L^T B^T) + \lambda^2(\epsilon_1 G_a G_a^T + \epsilon_2 G_b G_b^T + \epsilon_3 G_h G_h^T), \\
\Theta_{12} &= -X^T A^T - L^T B^T + \lambda X + G + \lambda(\epsilon_1 G_a G_a^T + \epsilon_2 G_b G_b^T + \epsilon_3 G_h G_h^T), \\
\Theta_{14} &= X^T C^T, \\
\Theta_{15} &= -X^T E_a^T, \\
\Theta_{16} &= -L^T E_b^T, \\
\Theta_{22} &= X + X^T + \epsilon_1 G_a G_a^T + \epsilon_2 G_b G_b^T + \epsilon_3 G_h G_h^T.
\end{aligned}$$

Then, the controller is given by $u = Kx = LX^{-1}x$.

Proof 3 *In order to consider uncertainties, let us replace A, B and H with $A + \Delta A(t)$, $B + \Delta B(t)$ and $H + \Delta H(t)$, respectively in (36) and thereby* *observe that*

$$\mathsf{He}\left\{\begin{bmatrix} -X^T\big(A+\Delta A(t)\big)^T - L^T\big(B+\Delta B(t)\big)^T \\ X^T \\ -(H+\Delta H(t))^T \\ 0 \end{bmatrix}\begin{bmatrix}\lambda I & I & 0 & 0\end{bmatrix}\right\} + \begin{bmatrix} 0 & G & 0 & X^TC^T \\ G & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & D^T \\ CX & 0 & D & -I \end{bmatrix} \prec 0. \quad (42)$$

Note that (42) *can be rewritten as,*

$$\Phi_n + \Phi_u \prec 0 \quad (43)$$

where

$$\Phi_n = \mathsf{He}\left\{\begin{bmatrix} -X^TA^T - L^TB^T \\ X^T \\ -H^T \\ 0 \end{bmatrix}\begin{bmatrix}\lambda I & I & 0 & 0\end{bmatrix}\right\} + \begin{bmatrix} 0 & G & 0 & X^TC^T \\ G & 0 & 0 & 0 \\ 0 & 0 & -\gamma_\infty^2 I & D^T \\ CX & 0 & D & -I \end{bmatrix},$$

and

$$\Phi_u = \mathsf{He}\left\{\underbrace{\begin{bmatrix}\lambda G_a \\ G_a \\ 0 \\ 0\end{bmatrix}}_{\mathcal{H}_1} F(t)\underbrace{\begin{bmatrix}-E_aX & 0 & 0 & 0\end{bmatrix}}_{\mathcal{E}_1}\right\}$$

$$+\,\mathsf{He}\left\{\underbrace{\begin{bmatrix}\lambda G_b\\ G_b\\ 0\\ 0\end{bmatrix}}_{\mathcal{H}_2} F(t)\,\underbrace{\begin{bmatrix}-E_bL & 0 & 0 & 0\end{bmatrix}}_{\mathcal{E}_2}\right\}$$

$$+\,\mathsf{He}\left\{\underbrace{\begin{bmatrix}\lambda G_h\\ G_h\\ 0\\ 0\end{bmatrix}}_{\mathcal{H}_3} F(t)\,\underbrace{\begin{bmatrix}0 & 0 & -E_h & 0\end{bmatrix}}_{\mathcal{E}_3}\right\}.$$

Then using Lemma 2, equation (43) turns into

$$\Phi_n+\epsilon_1\mathcal{H}_1\mathcal{H}_1^T+\epsilon_1^{-1}\mathcal{E}_1^T\mathcal{E}_1+\epsilon_2\mathcal{H}_2\mathcal{H}_2^T+\epsilon_2^{-1}\mathcal{E}_2^T\mathcal{E}_2+\epsilon_3\mathcal{H}_3\mathcal{H}_3^T+\epsilon_3^{-1}\mathcal{E}_3^T\mathcal{E}_3\prec 0. \quad (44)$$

Finally, applying Schur complement formulae on (44) gives (41). This concludes the proof.

Remark 2 *As it can be observed from Theorem 3, the controller is designed with an auxiliary variable instead of the symmetric Lyapunov matrix. This design process expands the solution set of LMIs. For this reason, dilated LMIs based robust controller synthesis methods provide less conservatism than the standard methods.*

4 Numerical simulation studies

In this part, the effectiveness of the proposed robust H_∞ controller is verified with numerical studies. Matlab and Simulink are used in all simulations and computations. For the solutions of the convex optimization problem that is described by LMIs, Yalmip Parser ([Löfberg(2004)]) and SeDuMi solver ([Sturm(1999)]) are used. The nominal values of the bilateral teleoperation system parameters are borrowed from [Du(2013)] and shown in Table 1.

Control outputs are selected as

$$z(t)=\begin{bmatrix}\frac{\int e_{t_m}(t)dt}{|\int e_{t_m}(t)dt|} & \frac{\int e_{t_s}(t)dt}{|\int e_{t_s}(t)dt|} & \frac{e_w(t)}{|e_w(t)|}\end{bmatrix}^T, \quad (45)$$

Table 1: System Parameters.

Parameter	Value	Unit
m_m	10	[kg]
m_s	10	[kg]
b_m	1	[Ns/m]
b_s	1	[Ns/m]
c_h	0.5	[Ns/m]
b_e	0.1	[Ns/m]
k_h	0.1	[N/m]
k_e	1	[N/m]

where $\int e_{t_m}(t)dt$ is the integral tracking error between the reference signal and the master and $|\int e_{t_m}(t)dt|$ denotes the estimated upper value of $\int e_{t_m}(t)dt$. On the other hand, $\int e_{t_s}(t)dt$ is an integral tracking error between the reference signal and the slave and $|\int e_{t_s}(t)dt|$ is its estimated upper value. $e_w(t) = \big(f_h(t) - f_e(t)\big)$ is the force error between the human operator and environmental-induced exogenous forces and $|e_w(t)|$ is the estimated upper value of $e_w(t)$. Here, control outputs can be defined as $\frac{\int e_{t_m}(t)dt}{|\int e_{t_m}(t)dt|} := \alpha \int e_{t_m}(t)dt$, $\frac{\int e_{t_s}(t)dt}{|\int e_{t_s}(t)dt|} := \delta \int e_{t_s}(t)dt$ and $\frac{e_w(t)}{|e_w(t)|} := \beta e_w(t)$ where α, δ and β are user-defined weights to be selected by obeying the condition $\alpha + \delta + \beta = 1$.

In order to provide a trade-off between control outputs, nonnegative weighting parameters α, δ, β are used. Thereby, controlled output matrices in (19) are chosen as follows:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{46}$$

and

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \beta & -\beta \end{bmatrix}. \tag{47}$$

Here, the minimization of $\int e_{t_m}(t)dt$ ensures that master follows the reference and the minimization of $\int e_{t_s}(t)dt$ enforces the slave to follow the reference. These requirements are needed for a satisfactory reference tracking. Finally, $e_w(t)$ is minimized to satisfy force reflection performance.

In the light of Theorem 2 and Theorem 3, Table 2 summarizes two different control problems, namely the standard robust H_∞ and its dilated version, for the open-loop uncertain bilateral teleoperation system (19).

Table 2: Controllers

Standard Robust H_∞	Dilated Robust H_∞
minimize γ_∞	minimize γ_∞^2
subject to (40), $Q \succ 0$	subject to (41), $G \succ 0$
controller gain $K = SQ^{-1}$	controller gain $K = LX^{-1}$

Case Study 1 We assumed that uncertainty bounds are i.e., $p_1 = p_2 = p_3 = p_4 = p = 0.55$, scalars at Lemma 2 are selected as $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 1.8$ and the weighting parameters are chosen as shown in Table 3.

Table 3: Weighting parameters in Case Study 1

Controller	α	δ	β
Standard Robust H_∞	0.3	0.64	0.06
Dilated Robust H_∞	0.3	0.64	0.06

Minimization problems summarized at Table 2 lead to feasible solutions, and γ and K are calculated as shown in Table 4 and Table 5, respectively.

Table 4: γ_∞ gains in Case Study 1

Controller	γ_∞
Standard Robust H_∞	9.1782
Dilated Robust H_∞	4.16

For the dilated robust H_∞ controller, the best value of λ is obtained as 10^4, through a line search that is performed using the γ_∞ gains obtained for different values of λ as indicated in Table 6.

Numerical results of the Case Study 1 are shown in Table 4, Table 5 and Table 6. Table 4 demonstrates that almost 2.5 times lower H_∞ norm can be obtained with the proposed dilated form compared to the standard

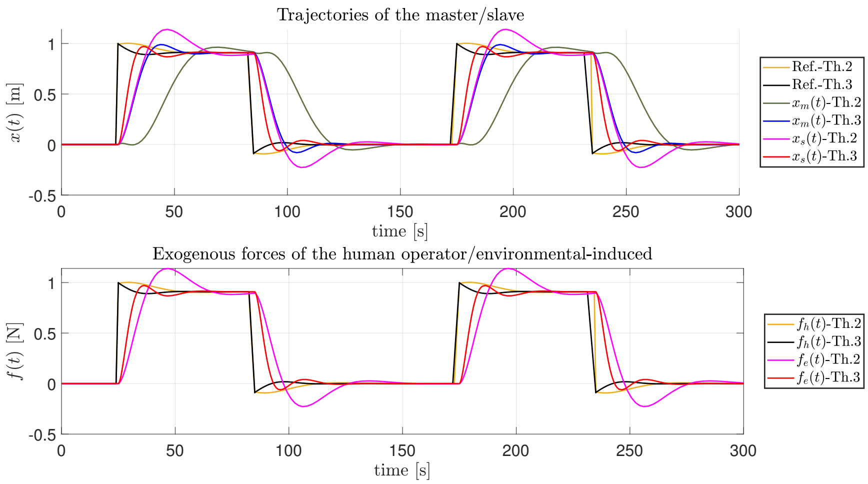

Figure 2: Reference tracking and force reflection in Case Study 1

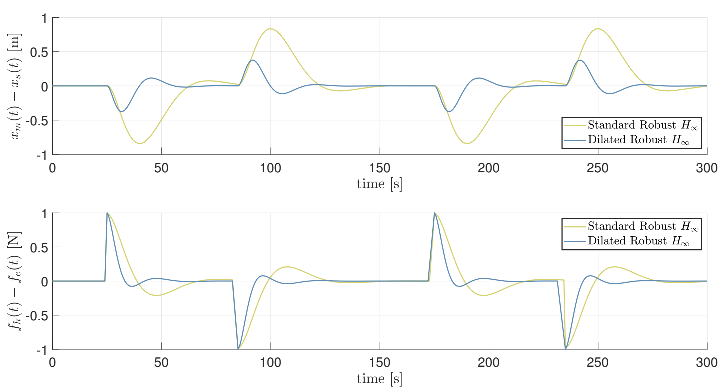

Figure 3: Tracking Errors in Case Study 1

Table 5: Controller gains in Case Study 1

Standard Robust H_∞
$10^7 \times \begin{bmatrix} -0.0334 & -0.6138 & 0.4187 & 1.3707 & -0.0075 & -0.0477 \\ 0.0745 & 1.3707 & -0.9350 & -3.0612 & 0.0168 & 0.1065 \end{bmatrix}$
Dilated Robust H_∞
$10^4 \times \begin{bmatrix} 0.0589 & -0.2505 & 0.5630 & 1.1473 & -0.0252 & -0.1093 \\ -0.2985 & 1.1474 & -2.7108 & -5.5240 & 0.1239 & 0.5256 \end{bmatrix}$

form. Table 5 indicates controller gains of controllers which are also used in simulations. According to the line search in Table 6, the value of λ that gives the minimum γ_∞ gain is used in the controller design.

Simulation results of the Case Study 1 are shown in Figure 2, Figure 3 and Figure 4. The top row of Figure 2 demonstrates trajectories of the master, the slave and the reference signal for both controllers. Here, the tracking performance between the master and the slave is more successful in the proposed dilated H_∞ controller than the standard H_∞ controller. One can observe that master/slave trajectories of the dilated controlled system are converging to the reference signal whereas the standard controlled system fails to do that. Moreover, the proposed controller has less overshoots than that of the standard controller. The bottom row of Figure 2 indicates exogenous forces of the human operator and the environmental-induced. Similarly, when it comes to the trajectories of the master/slave, the proposed controller performs much better in tracking exogenous forces. Furthermore, it produces less overshoots than the standard H_∞ controller. Figure 3 indicates tracking errors of master/slave trajectories and exogenous forces. Here, tracking errors indicates that dilated H_∞ controller produces less tracking errors both in reference tracking and force reflection than the standard method. Finally, Figure 4 shows control forces for Case Study 1 and indicates that the controller forces of both controllers are quite similar.

Case Study 2 We now assume that uncertainty bounds are i.e., $p_1 = p_2 = 0.3$ and $p_3 = p_4 = 0.86$ scalars at Lemma 2 are chosen as $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 1.6$ and the weighting parameters are given in Table 7.

Performing the minimization problems summarized at Table 2, we obtain the H_∞ gains shown in Table 8. On the other hand, associated controller gains for the dilated robust H_∞ and the standard robust H_∞ controllers are provided in Table 9.

Table 6: Line search used during the design of dilated robust H_∞ controller in Case Study 1

λ	γ_∞
2.3	*Infeasible*
2.4	20.59
2.5	12.98
5	5.03
10	4.46
50	4.21
100	4.19
10^3	4.17
10^4	4.16
10^5	4.16
10^6	4.17
$5 * 10^6$	15.62
10^7	29.78
$5 * 10^7$	*Infeasible*

Table 7: Weighting parameters in Case Study 2

Controller	α	β	δ
Standard Robust H_∞	0.3	0.62	0.08
Dilated Robust H_∞	0.3	0.62	0.08

For the dilated robust H_∞ controller, best performance is achieved for $\lambda = 10^4$. This value of λ is obtained by using a line search as indicated in Table 10.

Numerical results of the Case Study 2 are shown in Table 8, Table 9 and Table 10. Different from the Case Study 1, Table 8 demonstrates that almost 2.5 times lower closed-loop L_2 gain can be obtained with the usage of the proposed dilated form when compared to the standard form. Table 9 indicates controller gains of controllers which are also used in simulations. According to the line search in Table 10, λ that gives the minimum γ_∞ gain is chosen.

Simulation results of the Case Study 2 are shown in Figure 5, Figure 6 and Figure 7. The top row of Figure 5 demonstrates trajectories of the master,

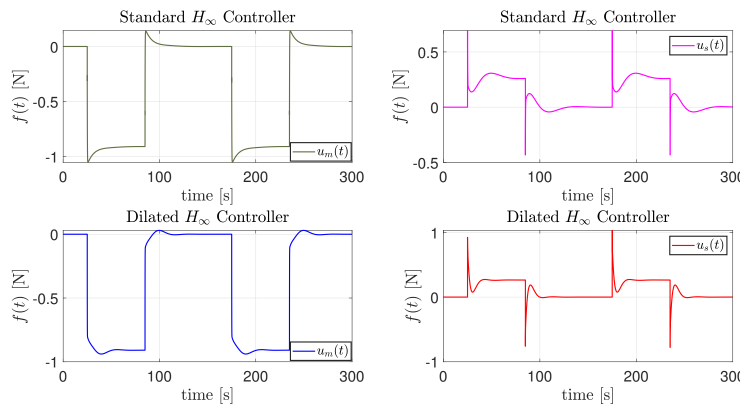

Figure 4: Control forces in Case Study 1

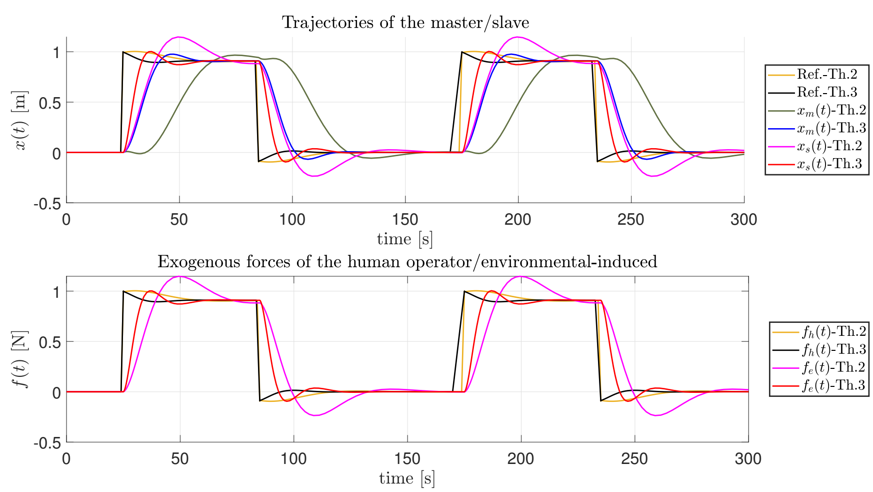

Figure 5: Reference tracking and force reflection in Case Study 2

Table 8: γ_∞ gains in Case Study 2

Controller	γ_∞
Standard Robust H_∞	10.9309
Dilated Robust H_∞	4.45

Table 9: Controller gains in Case Study 2

Standard Robust H_∞
$10^7 \times \begin{bmatrix} -0.0445 & -0.6897 & 0.4035 & 1.4692 & -0.0062 & -0.0404 \\ 0.0947 & 1.4692 & -0.8596 & -3.1299 & 0.0131 & 0.0860 \end{bmatrix}$
Dilated Robust H_∞
$10^4 \times \begin{bmatrix} 0.0625 & -0.3813 & 0.6915 & 1.5047 & -0.0292 & -0.1251 \\ -0.2640 & 1.5047 & -2.8046 & -6.1069 & 0.1199 & 0.5068 \end{bmatrix}$

the slave, and the reference signal for both controllers. The bottom row of Figure 5 shows exogenous forces of the human operator and the environment. Figure 6 shows the tracking errors of master/slave trajectories and exogenous forces. Here, the performance outputs are quite similar in Case Study 1. Finally, Figure 7 indicates control forces for Case Study 2 and indicates that controller forces at both controllers are quite similar values.

Consideration of norm bounded type parameter uncertainties in the process model, provides a more realistic simulation environment for the reference tracking and force reflection control of the bilateral teleoperation system. Numerical results show that almost 2.5 times lower H_∞ norm could be obtained by using the proposed dilated LMI form, compared to the standard form. Moreover, the proposed controller provides robustness for uncertain parameters m_m, m_s, b_m/b_s and b_s/m_s up to 55%. Also, the proposed method can provide robustness for different uncertain bounds such as 30%, 30%, 86% and 86% respectively for the same uncertain parameter values. The simulation results demonstrate that the proposed controller can perform successful reference tracking and force reflection, regardless of the very high parameter uncertainties on the system parameters.

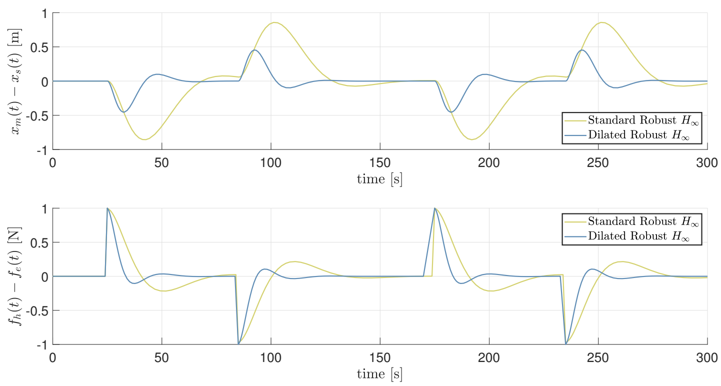

Figure 6: Tracking Errors in Case Study 2

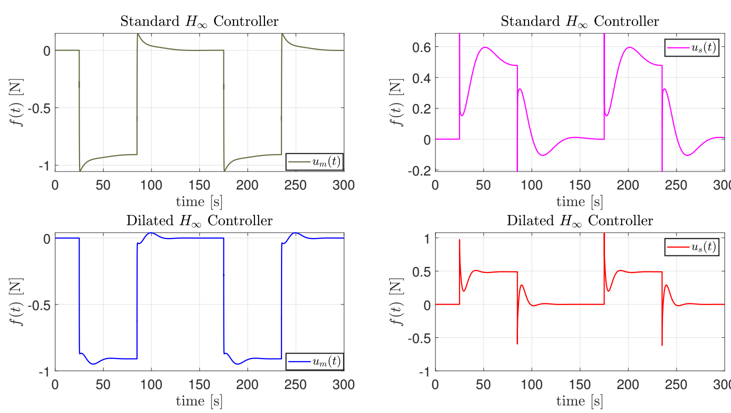

Figure 7: Control forces in Case Study 2

Table 10: Line search performed during the design of the dilated robust H_∞ controller in Case Study 2

λ	γ_∞
2.5	*Infeasible*
2.6	36.95
3	9.58
5	5.58
10	4.82
50	4.51
100	4.48
10^3	4.46
10^4	4.45
10^5	4.45
10^6	4.47
$5 * 10^6$	20.92
10^7	32.99
$3 * 10^7$	*Infeasible*

5 Conclusion

We considered the robust H_∞ controller design problem based on dilated LMIs for bilateral teleoperation systems having norm bounded parametric uncertainties. In this work, we developed a practically realizable control strategy to obtain a less conservative robust H_∞ controller based on state-feedback control law. Robust stability and performance requirements of the system have been satisfied with the Lyapunov Stability and the H_∞-norm from exogenous input to the controlled output, respectively. In order to demonstrate the effectiveness of the proposed controller, a one-degree-of-freedom uncertain master/slave system that includes a human operator exogenous force and an environmental-induced exogenous force has been used. Extensive numerical simulation studies have been performed in comparison with the standard robust H_∞ controller to reveal the effectiveness of the proposed controller. Numerical simulation studies showed that under norm bounded type uncertainties, the dilated robust H_∞ controller guaranteed robust closed-loop stability at lower γ_∞ gain, and provided better reference tracking and force reflection performance when compared to standard robust H_∞ controller.

For future works, the proposed dilated LMI method can be used with uncertainties represented in linear fractional representation form via modified full block S-procedure method. Our method is relies on the full state-feedback information which is restrictive. Therefore, it would be interesting to study output feedback control problem using dilated approach. Additionally, effects of time-delay over the communication channel between master and slave can also be considered.

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7 Appendix

Proof 4 *(Theorem 2) A bounded real-lemma can be written for the closed-loop system (35) as follows ([Boyd et al.(1994)]):*

If there exist matrices $W = W^T \succ 0$ and L such that

$$\begin{bmatrix} W(A+BK)+(A+BK)^T W & WH & C^T \\ * & -\gamma_\infty I & D^T \\ * & * & -\gamma_\infty I \end{bmatrix} \prec 0. \tag{48}$$

then H_∞ norm of (35) from its input w to output z is less than γ_∞.

After that, defining $Q := W^{-1}$, $S := KQ$ and applying a congruence transformation on (48) with a transformation matrix $\mathsf{diag}\{Q, I, I\}$ gives

$$\begin{bmatrix} AQ+QA^T+BS+S^T B^T & H & QC^T \\ * & -\gamma_\infty I & D^T \\ * & * & -\gamma_\infty I \end{bmatrix} \prec 0. \tag{49}$$

In order to consider uncertainties, let us replace A, B and H with $A+$

$\Delta A(t)$, $B+\Delta B(t)$ and $H+\Delta H(t)$, respectively in (49), which yields

$$\begin{bmatrix} \begin{array}{c}\big(A+\Delta A(t)\big)Q+Q\big(A+\Delta A(t)\big)^T \\ \qquad +\big(B+\Delta B(t)\big)S+S^T\big(B+\Delta B(t)\big)^T\end{array} & H+\Delta H(t) & QC^T \\ * & -\gamma_\infty I & D^T \\ * & * & -\gamma_\infty I \end{bmatrix} \prec 0. \tag{50}$$

Then, by decomposing the resulting matrix inequality into nominal and uncertain parts, one can obtain the following form:

$$\Pi_n + \Pi_u \prec 0 \tag{51}$$

where

$$\Pi_n = \begin{bmatrix} AQ+QA^T+BS+S^TB^T & H & QC^T \\ * & -\gamma_\infty I & D^T \\ * & * & -\gamma_\infty I \end{bmatrix},$$

and

$$\Pi_u = \mathsf{He}\left\{ \underbrace{\begin{bmatrix} G_a \\ 0 \\ 0 \end{bmatrix}}_{\mathcal{H}_4} F(t) \underbrace{\begin{bmatrix} E_aQ & 0 & 0 \end{bmatrix}}_{\mathcal{E}_4} \right\}$$

$$+\mathsf{He}\left\{ \underbrace{\begin{bmatrix} G_b \\ 0 \\ 0 \end{bmatrix}}_{\mathcal{H}_5} F(t) \underbrace{\begin{bmatrix} E_bS & 0 & 0 \end{bmatrix}}_{\mathcal{E}_5} \right\} + \mathsf{He}\left\{ \underbrace{\begin{bmatrix} G_h \\ 0 \\ 0 \end{bmatrix}}_{\mathcal{H}_6} F(t) \underbrace{\begin{bmatrix} 0 & E_h & 0 \end{bmatrix}}_{\mathcal{E}_6} \right\}.$$

Based on the Lemma 2, inequality (51) turns into

$$\Pi_n + \epsilon_4\mathcal{H}_4\mathcal{H}_4^T + \epsilon_4^{-1}\mathcal{E}_4^T\mathcal{E}_4 + \epsilon_5\mathcal{H}_5\mathcal{H}_5^T + \epsilon_5^{-1}\mathcal{E}_5^T\mathcal{E}_5 + \epsilon_6\mathcal{H}_6\mathcal{H}_6^T + \epsilon_6^{-1}\mathcal{E}_6^T\mathcal{E}_6 \prec 0. \tag{52}$$

Finally, applying Schur complement formulae on (52) gives (40). This concludes the proof.

References

[Aktas et al.(2018)] Aktas A, Yazici H and Sever M (2018) LMI-based design of an I-PD+PD type LPV state feedback controller for a gantry crane. *Transactions of the Institute of Measurement and Control*, 41(6): 1640–1655.

[Apkarian et al.(2001)] Apkarian P, Tuan HD and Bernussou J (2001) Continuous-time analysis, eigenstructure assignment, and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations. *IEEE Transactions on Automatic Control*, 46(12): 1941–1946.

[Beikzadeh and Marquez(2018)] Beikzadeh H and Marquez HJ (2018) Robust Sampled-Data Bilateral Teleoperation: Single-Rate and Multirate Stabilization. *IEEE Transactions on Control of Network Systems* 5(1): 675–687.

[Boyd et al.(1994)] Boyd S, El Gahoui L, Feron E et al. (1994) *Linear Matrix Inequalities in System and Control Theory*, vol. 15 of SIAM Studies in Applied Mathematics, SIAM.

[De Oliveira et al.(2002)] De Oliveira MC, Geromel JC and Bernussou J (2002) Extended H_2 and H_∞ norm characterizations and controller parametrizations for discrete-time systems. *International Journal of Control*, 75(9): 666–679.

[Du(2013)] Du H (2013) H_∞ state-feedback control of bilateral teleoperation systems with asymmetric time-varying delays. *IET Control Theory and Applications* 7(4): 594–605.

[Ebihara and Hagiwara(2004)] Ebihara Y and Hagiwara T (2004) New dilated LMI characterizations for continuous-time multiobjective controller synthesis. *Automatica* 40(11): 2003–2009.

[Ferrell(1965)] Ferrell MW (1965) Remote manipulation with transmission delay. *IEEE Transactions on Human Factors in Electronics* HFE-6(1): 24–32.

[Gahinet and Apkarian(1994)] Gahinet P and Apkarian P (1994) A linear matrix inequality approach to H_∞ control. *International Journal of Robust and Nonlinear Control*, 4(4): 421–448.

[Goertz(1949)] Goertz RC (1949) *Master-Slave Manipulator*. Illinois, USA: Argonne National Laboratory.

[Hokayem and Spong(2006)] Hokayem PF and Spong MW (2006) Bilateral teleoperation: An historical survey. *Automatica* 42(12): 2035–2057.

[Lawrence(1993)] Lawrence DA (1993) Stability and transparency in bilateral teleoperation. *IEEE Transactions on Robotics and Automation* 9(5): 624–637.

[Leung et al.(1995)] Leung GMH, Francis BA, Apkarian J (1995) Bilateral controller for teleoperators with time delay via μ-synthesis. *IEEE Transactions on Robotics and Automation* 11(1): 105–116.

[Löfberg(2004)] Löfberg J (2004) YALMIP : a toolbox for modeling and optimization in MATLAB. In: *Proceedings of 2004 IEEE International Conference on Robotics and Automation*, Taipei, Taiwan, pp. 284–289. IEEE.

[Miyazaki et al.(1986)] Miyazaki F, Matsubayashi S, Yoshimi T et al. (1986) A new control methodology toward advanced teleoperation of master-slave robot systems. In: *Proceedings of 1986 IEEE International Conference on Robotics and Automation*, San Francisco, CA, USA, pp. 997–1002. IEEE.

[Parlakçı(2006)] Parlakçı MNA (2006) Improved robust stability criteria and design of robust stabilizing controller for uncertain linear time-delay systems. *International Journal of Robust and Nonlinear Control* 16(13): 599-–636.

[Pipeleers et al.(2009)] Pipeleers G, Demeulenaere B, Swevers J et al. (2009) Extended LMI characterizations for stability and performance of linear systems. *Systems & Control Letters*, 58(7): 510–518.

[Sajjadi-Kia and Jabbari(2007)] Sajjadi-Kia S and Jabbari F (2007) Dilated matrix inequalities for control design in systems with actuator constraint. In: *Proceedings of 46th IEEE Conference on Decision and Control*, New Orleans, LA, USA, pp. 5678–5683. IEEE.

[Sheridan and Ferrell(1963)] Sheridan PF and Ferrell MW (1963) Remote Manipulative Control with Transmission Delay. *IEEE Transactions on Human Factors in Electronics* HFE-4(1): 25–29.

[Sturm(1999)] Sturm JF (1999) Using SeDuMi 1.02, A Matlab toolbox for optimization over symmetric cones. *Optimization Methods and Software*, 11(1-4): 625–653.

[Xie(1996)] Xie L (1996) Output feedback H_∞ control of systems with parameter uncertainty. *International Journal of Control* 63(4): 741–750.

[Yang et al.(2015)] Yang Y, Li H, Chen Y et al. (2015) Model predictive control for space teleoperation systems based on a mixed-H_2/H_∞ approach. *Journal of Aerospace Engineering*, 28(5): 04014133-1–04014133-9.

[Zhang et al.(2012)] Zhang B, Kruszewski A and Richard JP (2012) H_∞ robust control design for teleoperation systems. In: *Proceedings of 7th IFAC Symposium on Robust Control Design*, Aalborg, Denmark, June 20-22, pp. 666–671. IFAC.