Delay-dependent guaranteed cost gain scheduling control of LPV state-delayed systems

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Motivation

Delays needs to be considered in engineering systems because

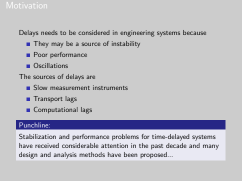
- They may be a source of instability
- Poor performance
- Oscillations

The sources of delays are

- Slow measurement instruments
- Transport lags
- Computational lags

Punchline:

Stabilization and performance problems for time-delayed systems have received considerable attention in the past decade and many design and analysis methods have been proposed...



Of course many things required to be done. We will talk about this at the end.

Classification of Delays..

The existing research work

on delayed systems can be divided into two main categories

- delay independent methods Very conservative !!!
- delay dependent methods Less conservative compared to the delay independent one !!!

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Delay independent controllers are designed to stabilize the system irrespective of the size of the delay,
may not stabilize some kind of delayed systems or may require very conservative control schemes

Different from the delay independent counterparts, delay dependent controllers serve much more powerful and less conservative controllers

From stability to Performance...

One approach is to guarantee a level of performance from the designed system. Named as :Guaranteed Cost Control

Performance measure

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Performance measure

Generally, the performance measure is chosen as a linear quadratic cost function of system states and inputs.

Stability is not only problem to be solved
It is also desirable to have control system which is not only stable but also guarantees an adequate level of performance

provides an upper bound on the performance measure and thus the system performance degradation incurred by the delays is guaranteed to be less than an upper bound In the last decade this approach has been greatly developed and many significant contributions have been performed in the literature

Some features of the literature

- They are mostly related with the norm bounded uncertainties
- LPV systems with delay are NOT common
- Time-varying delays are NOT common

Motivated by these issues

A delay dependent gain scheduling design problem for guaranteed cost control of state delayed LPV systems with time varying delays is considered...

Presented method is based on

Lyapunov-Krasovskii functionals (as usual) in terms of MIs which leads to a Non-Convex optimization problem :(
However a method named as cone-complementarity algorithm is adapted to our problem to solve the non-convex problem in terms of LMIs (Thanks to El-Ghaoui)

Problem Formulation...

System

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + A_1(\theta)x(t-h(t)) + Bu(t) \\ x(t) &= \phi(t), \qquad t \in [-\bar{h}, 0] \\ u(t) &= K(\theta)x(t) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, A and $A_1 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ $u(t) \in \mathbb{R}^m$ is the control vector, $\phi(t) \in \mathcal{C}_0$ is the initial condition, $h(t)$ is the time-varying delay which satisfies $|\dot{h}(t)| \leq \bar{h}_v < 1$ and $0 \leq h(t) \leq \bar{h}$, for all $t \geq 0$, $\theta \in \mathbb{R}^p$, is the programming variable vector assumed to be measurable during the run-time and θ varies in a polytope Θ of vertices $\theta_1, \theta_2, \ldots, \theta_r$ where $r = 2^p$.

Problem Formulation...

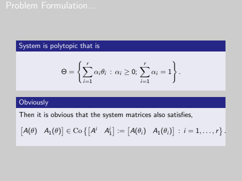
System is polytopic that is

$$\Theta = \left\{ \sum_{i=1}^{r} \alpha_i \theta_i \,:\, \alpha_i \geq 0; \sum_{i=1}^{r} \alpha_i = 1 \right\}.$$

Obviously

Then it is obvious that the system matrices also satisfies,

$$\begin{bmatrix} A(\theta) & A_1(\theta) \end{bmatrix} \in \mathrm{Co}\left\{ \begin{bmatrix} A^i & A_1^i \end{bmatrix} := \begin{bmatrix} A(\theta_i) & A_1(\theta_i) \end{bmatrix} \,:\, i = 1, \ldots, r \right\}.$$



it is a very well known property of polytopic convex sets.

Indeed, when Θ is not a polytope, the results developed herein can still be applied by replacing Θ with some polytope $\Theta_{\text{poly}} \supset \Theta$.

Problem Formulation...

Cost Function

[label=cost]

$$J = \int_0^{\infty} \left\{ x^{\mathrm{T}}(t) Q x(t) + u^{\mathrm{T}}(t) R u(t) \right\} dt,$$

where $t \in \mathbb{R}_e$, $Q > 0$ and $R > 0$.

Definition (G.C.C)

Consider the plant and the control signal $u(t)$ in the plant. If there exist a gain scheduled control $u(\theta)$ and $\bar{J}$ such that, for all admissible system trajectories, the closed-loop system stays stable and the cost function always satisfies $J \leq \bar{J}$, then $\bar{J}$ is said to be a guaranteed cost for the plant and $u(\theta)$ is said to be the guaranteed cost, gain-scheduled controller for the LPV system.

Problem Formulation...

Control Problem

The control problem is to find a suitable memoryless gain-scheduled guaranteed cost control law $u = K(\theta)x$ which minimizes the cost function

$$J = \int_0^\infty \left\{ x^{\mathrm{T}}(t) Q x(t) + u^{\mathrm{T}}(t) R u(t) \right\} dt,$$

by minimizing an upper bound $\bar{J}$.

Problem Formulation...

Control Problem

Since θ varies in a polytope the controller can be defined as

$$K(\theta) = \sum_{i=1}^{r} \alpha_i K^i,$$

where K^i is the controller that is associated with the i-th vertex of the convex polytope Θ and $(\alpha_1, \ldots, \alpha_r)$ is any solution of the convex decomposition problem

$$\theta = \sum_{i=1}^{r} \alpha_i \theta_i.$$

Problem Formulation...

Closed Loop System Equation

$$
\begin{aligned}
\dot{x}(t) &= [A(\theta) + BK(\theta)]x(t) + A_1(\theta)x(t - h(t)) \\
x(t) &= \phi(t), \qquad t \in [-\bar{h}, 0]
\end{aligned}
$$

Some useful MI Identities

Indentity #1

$$-2a^T b \leq \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - I \\ Y^{\mathrm{T}} - I & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

where

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0.$$

Indentity #2

$$-2 \int_{\Omega} a^{\mathrm{T}}(\sigma)\, \Phi\, b(\sigma) \mathrm{d}\sigma \leq \int_{\Omega} \begin{bmatrix} a(\sigma) \\ b(\sigma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y - \Phi \\ (Y - \Phi)^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} a(\sigma) \\ b(\sigma) \end{bmatrix} \mathrm{d}\sigma$$

where

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0.$$

Design of the gain-scheduling controller with guaranteed

Theorem

Consider the closed-loop system with the cost defined function .
$u(t) = K(\theta)x(t)$ *is a guaranteed cost controller if there exist* $P > 0$, S, X, Y, Z *in appropriate dimensions such that*

$$\begin{bmatrix} \Sigma & PA_1(\theta) - Y & \bar{h}[A(\theta) + BK(\theta)]^{\mathrm{T}} Z \\ \star & (\bar{h}_v - 1)S & \bar{h} A_1^{\mathrm{T}}(\theta) Z \\ \star & \star & -\bar{h} Z \end{bmatrix} < 0,$$

where
$\Sigma = P[A(\theta) + BK(\theta)] + [A(\theta) + BK(\theta)]^{\mathrm{T}} P + \bar{h}X + Y + Y^{\mathrm{T}} + Q + K^{\mathrm{T}} RK + S$
and

$$\begin{bmatrix} X & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geq 0$$

for every $\theta \in \Theta$.

Corollary

Corollary

Then the closed-loop system is asymptotically stable for any time delay $h(t)$ that satisfies $0 \leq h(t) \leq \bar{h}$ and $\dot{h}(t) \leq \bar{h}_v < 1$. Furthermore, the cost function J satisfies the following inequality

$$J \leq \phi^{\mathrm{T}}(0)P\phi(0) + \int_{-\bar{h}}^{0}\int_{\beta}^{0} \dot{x}^{\mathrm{T}}(\sigma)Z\dot{x}(\sigma)d\sigma d\beta + \int_{-\bar{h}}^{0} x^{\mathrm{T}}(\sigma)Sx(\sigma)\mathrm{d}\sigma \triangleq \bar{J}.$$

Sketch of the proof

Choose a candidate Lyapunov Equation V s.t. $V = V_1 + V_2 + V_3$ where

$$V_1 \triangleq x^{\mathrm{T}}(t)Px(t),$$

$$V_2 \triangleq \int_{-\bar{h}}^{0}\int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\sigma)Z\dot{x}(\sigma)\mathrm{d}\sigma\mathrm{d}\beta,$$

$$V_3 \triangleq \int_{t-h(t)}^{t} x^{\mathrm{T}}(\sigma)Sx(\sigma)\mathrm{d}\sigma.$$

Sketch of the Proof (cont.)

cont...

Then use the MI identities we gave before and the property

$$x(t) - x(t - h(t)) = \int_{t-h(t)}^{t} \dot{x}(\sigma)\mathrm{d}\sigma.$$

and take the time derivative of V w/t the system trajectory and making use of Leibnitz Integral formula $\rightarrow$

$$\begin{aligned}\dot{V} \leq \; & x^{\mathrm{T}}(t)[(A(\theta) + BK(\theta))^{\mathrm{T}}P + P(A(\theta) + BK(\theta)) \\ & + \bar{h}X + Y + Y^{\mathrm{T}} + \bar{h}(A(\theta) + BK(\theta))^{\mathrm{T}}Z(A(\theta) + BK(\theta)) + S]x(t) \\ & + 2x^{\mathrm{T}}(t)[PA_1(\theta) - Y + \bar{h}(A(\theta) + BK(\theta))^{\mathrm{T}}ZA_1(\theta)]x(t - h(t)) \\ & + x^{\mathrm{T}}(t - h(t))[(\bar{h}_v - 1)S + \bar{h}A_1^{\mathrm{T}}(\theta)ZA_1(\theta)]x(t - h(t)).\end{aligned}$$

This is nothing but the MI given in the theorem. If the inequality holds true for every θ, then one can conclude that

$$\dot{V} \leq -x^{\mathrm{T}}(t)(Q + K(\theta)^{\mathrm{T}}RK(\theta))x(t) < 0. \quad (1)$$

Hence, the closed-loop system is asymptotically stable if matrix inequalities in the theorem hold for every $\theta \in \Theta$.

Sketch of the Proof (cont.)

cont...

In order to show the least upper bound on the performance J, integrate both sides of

$$\dot{V} \leq -x^{\mathrm{T}}(t)(Q + K(\theta)^{\mathrm{T}} R K(\theta)) x(t).$$

from 0 to ∞.

Remark

Although the matrix inequalities in Theorem are LMIs when $A(\theta)$, $A_1(\theta)$, B, $\bar{h}$ and $K(\theta)$ are known, it can not be used in its present form for synthesis of the gain-scheduled control $K(\theta)$.

Controller Design

Theorem

If there exist matrices $\mathcal{P} > 0$, $\mathcal{K}^i$, $\mathcal{X}$, $\mathcal{Z} > 0$, $\mathcal{Y}$ and $\mathcal{S}$ in appropriate dimensions such that

$$\begin{bmatrix} A^i\mathcal{P}+B\mathcal{K}^i+\mathcal{P}A^{i\mathrm{T}}+\mathcal{K}^{i\mathrm{T}}B^{\mathrm{T}}+\bar{h}\mathcal{X}+\mathcal{Y}+\mathcal{Y}^{\mathrm{T}}+\mathcal{S} & A_1^i\mathcal{P}-\mathcal{Y} & \bar{h}\mathcal{P}A^{i\mathrm{T}}+\bar{h}\mathcal{K}^{i\mathrm{T}}B^{\mathrm{T}} & \mathcal{P} & \mathcal{K}^{i\mathrm{T}} \\ \star & (\bar{h}_v-1)\mathcal{S} & \bar{h}\mathcal{P}A_1^{i\mathrm{T}} & 0 & 0 \\ \star & \star & -\bar{h}\mathcal{Z} & 0 & 0 \\ \star & \star & \star & -Q^{-1} & 0 \\ \star & \star & \star & \star & -R^{-1} \end{bmatrix} < 0,$$

and

$$\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \star & \mathcal{P}\mathcal{Z}^{-1}\mathcal{P} \end{bmatrix} \geq 0, \qquad i = 1, \ldots, r.$$

$\mathcal{P} \triangleq P^{-1}$, $\mathcal{K} \triangleq KP^{-1}$, $\mathcal{X} \triangleq P^{-1}XP^{-1}$, $\mathcal{Y} \triangleq P^{-1}YP^{-1}$, $\mathcal{S} \triangleq P^{-1}SP^{-1}$ *and* $\mathcal{Z} \triangleq Z^{-1}$ *Then the closed-loop system is uniformly asymptotically stable for any $h(t)$ satisfying $0 \leq h(t) \leq \bar{h}$ and $\dot{h}(t) \leq \bar{h}_v < 1$ with the control signal*

$$u(t) = \sum_{i=1}^{r} \alpha_i \mathcal{P}\mathcal{K}^i,$$

where $(\alpha_1, \ldots, \alpha_r)$ is any solution of the convex decomposition problem

$$\theta = \sum_{i=1}^{r} \alpha_i \theta_i.$$

cont'...

Moreover, the cost function J satisfies

$$J \le \phi^{\mathrm{T}}(0)\mathcal{P}^{-1}\phi(0) + \int_{-\bar{h}}^{0}\int_{\beta}^{0} \dot{x}^{\mathrm{T}}(\sigma)\mathcal{Z}^{-1}\dot{x}(\sigma)\mathrm{d}\sigma\mathrm{d}\beta + \int_{-\bar{h}}^{0} x^{\mathrm{T}}(\sigma)\mathcal{P}^{-1}\mathcal{S}\mathcal{P}^{-1}x(\sigma)\mathrm{d}\sigma = \bar{J}.$$

Notice that the second inequality is not convex!!!

To overcome the non-convexity problem

use

$$\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \mathcal{Y}^{\mathrm{T}} & \Phi \end{bmatrix} \ge 0, \qquad \begin{bmatrix} \Phi^{-1} & \mathcal{P}^{-1} \\ \mathcal{P}^{-\mathrm{T}} & \mathcal{Z}^{-1} \end{bmatrix} \ge 0. \tag{2}$$

By introducing the new variables $\mathcal{M} \triangleq \mathcal{Z}^{-1}$, $\mathcal{N} \triangleq \mathcal{P}^{-1}$ and $\mathcal{H} \triangleq \Phi^{-1}$, this condition can be replaced by

$$\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \mathcal{Y}^{\mathrm{T}} & \Phi \end{bmatrix} \ge 0, \qquad \begin{bmatrix} \mathcal{H} & \mathcal{N} \\ \mathcal{N}^{\mathrm{T}} & \mathcal{M} \end{bmatrix} \ge 0.$$

Upper bound on the Performance Index

In order to minimize the upper bound of J, let us introduce the following matrix functions:

$$\Gamma_1 \triangleq \phi(0)\phi^{\mathrm{T}}(0),$$

$$\Gamma_2 \triangleq \int_{-\bar{h}}^{0}\int_{\beta}^{0} \dot{x}(\sigma)\dot{x}^{\mathrm{T}}(\sigma)\mathrm{d}\sigma\mathrm{d}\beta,$$

$$\Gamma_3 \triangleq \int_{-\bar{h}}^{0} x(\sigma)x^{\mathrm{T}}(\sigma)\mathrm{d}\sigma.$$

The upper bound γ on the first term in the right side of cost is defined by

$$\phi^{\mathrm{T}}(0)\mathcal{P}^{-1}\phi(0) < \gamma.$$

Upper bound on J cont.

using the Shur complement, bound on first term of the Lyapunov equation can be replaced with

$$\begin{bmatrix} \gamma & \phi^{\mathrm{T}}(0) \\ \phi(0) & \mathcal{P} \end{bmatrix} > 0.$$

Also, the second term of the Lyapunov equation can be written as

$$\int_{-\bar{h}}^{0} \int_{\beta}^{0} \dot{x}^{\mathrm{T}}(\sigma) \mathcal{Z}^{-1} \dot{x}(\sigma) \mathrm{d}\sigma \mathrm{d}\beta = \mathrm{tr}(\Gamma_2 \mathcal{Z}^{-1}) = \mathrm{tr}(\Gamma_2^{1/2} \mathcal{Z}^{-1} \Gamma_2^{1/2}).$$

Introduce a new variable $\Upsilon = \Upsilon^{\mathrm{T}}$ such that $\Gamma_2^{1/2} \mathcal{Z}^{-1} \Gamma_2^{1/2} < \Upsilon$ holds. Using the Shur complement bound on the second term is equivalent to

$$\begin{bmatrix} \Upsilon & \Gamma_2^{1/2} \\ \star & \mathcal{Z} \end{bmatrix} > 0.$$

bound on 3th term of Lyapunov function

Finally, to derive an upper bound on the third term, let us introduce $\Delta = \Delta^{\mathrm{T}}$ such that

$$\mathcal{P}^{-1}\mathcal{S}\mathcal{P}^{-1} < \Delta,$$

holds. By using the Shur complement, this is equivalent to

$$\begin{bmatrix} \Delta & \mathcal{P}^{-1} \\ \mathcal{P}^{-\mathrm{T}} & \mathcal{S}^{-1} \end{bmatrix} > 0.$$

Introducing the new variable $\mathcal{T} \triangleq \mathcal{S}^{-1}$,this can be replaced with

$$\begin{bmatrix} \Delta & \mathcal{N} \\ \mathcal{N}^{\mathrm{T}} & \mathcal{T} \end{bmatrix} > 0.$$

bound on 3th term of Lyapunov function

Then it is obvious that

$$\int_{-\bar{h}}^{0} x^{\mathrm{T}}(\sigma)\mathcal{P}^{-1}\mathcal{S}\mathcal{P}^{-1}x(\sigma)\mathrm{d}\sigma < \mathrm{tr}(\Gamma_3\Delta).$$

Besides, for a constant $\delta > 0$, we can assume

$$\gamma + \mathrm{tr}(\Upsilon) + \mathrm{tr}(\Gamma_3\Delta) < \delta.$$

Optimization problem

By combining all these constraints, one can obtain the following nonlinear optimization procedure for the guaranteed cost gain-scheduling controller:

Nonconvex problem

$$\min_{\mathcal{P},\mathcal{K}^i,\mathcal{X},\mathcal{Y},\mathcal{S},\mathcal{Z},\mathcal{M},\mathcal{N},\mathcal{H},\Delta,\Phi,\mathcal{T},\Upsilon,\gamma} \delta$$

$$\text{subject to}$$

$$\begin{bmatrix} A^i\mathcal{P}+B\mathcal{K}^i+\mathcal{P}A^{i^\mathrm{T}}+\mathcal{K}^{i^\mathrm{T}}B^\mathrm{T}+\bar{h}\mathcal{X}+\mathcal{Y}+\mathcal{Y}^\mathrm{T}+\mathcal{S} & A_1^i\mathcal{P}-\mathcal{Y} & \bar{h}\mathcal{P}A^{i^\mathrm{T}}+\bar{h}\mathcal{K}^{i^\mathrm{T}}B^\mathrm{T} & \mathcal{P} & \mathcal{K}^{i^\mathrm{T}} \\ \star & (\bar{h}_v-1)\mathcal{S} & \bar{h}\mathcal{P}A_1^{i^\mathrm{T}} & 0 & 0 \\ \star & \star & -\bar{h}\mathcal{Z} & 0 & 0 \\ \star & \star & \star & -Q^{-1} & 0 \\ \star & \star & \star & \star & -R^{-1} \end{bmatrix} < 0,$$

and

$$\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \mathcal{Y}^\mathrm{T} & \Phi \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \mathcal{H} & \mathcal{N} \\ \mathcal{N}^\mathrm{T} & \mathcal{M} \end{bmatrix} \geq 0.$$

$$\begin{bmatrix} \gamma & \phi^\mathrm{T}(0) \\ \phi(0) & \mathcal{P} \end{bmatrix} > 0.$$

$$\begin{bmatrix} \Upsilon & \Gamma_2^{1/2} \\ \star & \mathcal{Z} \end{bmatrix} > 0.$$

$$\begin{bmatrix} \Delta & \mathcal{N} \\ \mathcal{N}^\mathrm{T} & \mathcal{T} \end{bmatrix} > 0.$$

$$\gamma + \mathrm{tr}(\Upsilon) + \mathrm{tr}(\Gamma_3\Delta) < \delta.$$

Remark!!!

Notice that, due to the existence of the terms $\mathcal{Z}^{-1} = \mathcal{M}$, $\mathcal{P}^{-1} = \mathcal{N}$, $\Phi^{-1} = \mathcal{H}$ and $\mathcal{S}^{-1} = \mathcal{T}$, this problem is nonconvex.

Cone-Complementarity Algorithm

However, by using the cone complementarity linearization algorithm that is proposed in El-Ghaoui, one can search for a feasible solution for this non-convex optimization problem.

Problem in the format of Cone-Complementarity

$$\min_{\mathcal{P},\mathcal{K}^i,\mathcal{X},\mathcal{Y},\mathcal{S},\mathcal{Z},\mathcal{M},\mathcal{N},\mathcal{H},\Delta,\Phi,\mathcal{T},\Upsilon,\gamma} \mathrm{tr}(\mathcal{Z}\mathcal{M}+\mathcal{P}\mathcal{N}+\Phi\mathcal{H}+\mathcal{S}\mathcal{T})$$
$$\text{subject to}$$

$$\begin{bmatrix} A^i\mathcal{P}+B\mathcal{K}^i+\mathcal{P}A^{i^{\mathrm{T}}}+\mathcal{K}^{i^{\mathrm{T}}}B^{\mathrm{T}}+\bar{h}\mathcal{X}+\mathcal{Y}+\mathcal{Y}^{\mathrm{T}}+\mathcal{S} & A_1^i\mathcal{P}-\mathcal{Y} & \bar{h}\mathcal{P}A^{i^{\mathrm{T}}}+\bar{h}\mathcal{K}^{i^{\mathrm{T}}}B^{\mathrm{T}} & \mathcal{P} & \mathcal{K}^{i^{\mathrm{T}}} \\ \star & (\bar{h}_v-1)\mathcal{S} & \bar{h}\mathcal{P}A_1^{i^{\mathrm{T}}} & 0 & 0 \\ \star & \star & -\bar{h}\mathcal{Z} & 0 & 0 \\ \star & \star & \star & -Q^{-1} & 0 \\ \star & \star & \star & \star & -R^{-1} \end{bmatrix} < 0,$$

and

$$\begin{bmatrix} \mathcal{X} & \mathcal{Y} \\ \mathcal{Y}^{\mathrm{T}} & \Phi \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \mathcal{H} & \mathcal{N} \\ \mathcal{N}^{\mathrm{T}} & \mathcal{M} \end{bmatrix} \geq 0.$$

$$\begin{bmatrix} \gamma & \phi^{\mathrm{T}}(0) \\ \phi(0) & \mathcal{P} \end{bmatrix} > 0.$$

$$\begin{bmatrix} \Upsilon & \Gamma_2^{1/2} \\ \star & \mathcal{Z} \end{bmatrix} > 0.$$

$$\begin{bmatrix} \Delta & \mathcal{N} \\ \mathcal{N}^{\mathrm{T}} & \mathcal{T} \end{bmatrix} > 0.$$

$$\gamma + \mathrm{tr}(\Upsilon) + \mathrm{tr}(\Gamma_3\Delta) < \delta.$$

and $\begin{bmatrix} \mathcal{Z} & I \\ I & \mathcal{M} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \mathcal{P} & I \\ I & \mathcal{N} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \Phi & I \\ I & \mathcal{H} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \mathcal{S} & I \\ I & \mathcal{T} \end{bmatrix} \geq 0, \quad \forall i = 1, \ldots, r.$

Conclusion remarks

Best result to be obtained:

If one finds $\mathrm{tr}(\mathcal{Z}\mathcal{M} + \mathcal{P}\mathcal{N} + \Phi\mathcal{H} + \mathcal{S}\mathcal{T}) = 4n$ then the control rule asymptotically stabilize the delayed-system.
In order to accomplish this, an algorithm so-called CC Algorithm has been developed. This algorithm is as follows:

- Choose a sufficiently large δ such that there exist a feasible solution to the problem and set $\delta_{so} = \delta$. If can not find such a δ then exit.
- Find the feasible set $(\mathcal{Z}_0, \mathcal{M}_0, \mathcal{P}_0, \mathcal{N}_0, \Phi_0, \mathcal{H}_0, \mathcal{S}_0, \mathcal{T}_0)$ satisfying LMIs and set $k = 0$.
- Solve the following LMI problem for the variable set $(\mathcal{Z}, \mathcal{M}, \mathcal{P}, \mathcal{N}, \Phi, \mathcal{H}, \mathcal{S}, \mathcal{T})$:

  $$\mathrm{Min}\ \mathrm{tr}(\mathcal{Z}_\mathrm{k}\mathcal{M} + \mathcal{P}_\mathrm{k}\mathcal{N} + \Phi_\mathrm{k}\mathcal{H} + \mathcal{S}_\mathrm{k}\mathcal{T} + \mathcal{M}_\mathrm{k}\mathcal{Z} + \mathcal{N}_\mathrm{k}\mathcal{P} + \mathcal{H}_\mathrm{k}\Phi + \mathcal{T}_\mathrm{k}\mathcal{S})$$
  subject to the LMI constraints defined above

  Set $\mathcal{Z}_{k+1} = \mathcal{Z}, \mathcal{M}_{k+1} = \mathcal{M}, \mathcal{P}_{k+1} = \mathcal{P}, \mathcal{N}_{k+1} = \mathcal{N}, \Phi_{k+1} = \Phi, \mathcal{H}_{k+1} = \mathcal{H}, \mathcal{S}_{k+1} = \mathcal{S}, \mathcal{T}_{k+1} = \mathcal{T}$
- If some stopping conditions are met, then set $\delta_{so} = \delta$ and decrease δ for a pre-defined value and go to Step 2. Otherwise, If these conditions are not met within pre-specified number of iterations $(k_{\max})$, then exit. Else, set $k = k + 1$ and go to Step 3.

Numerical Example

C

onsider the following linear time-varying state delayed system:

$$\dot{x} = \begin{bmatrix} 0 & 1 + c\sin t \\ -2 & -3 + p\sin t \end{bmatrix} x(t) + \begin{bmatrix} c\sin t & 0.1 \\ -0.2 + p\sin t & -0.3 \end{bmatrix} x(t - \mu|\cos(\omega t)|) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t),$$

where $\phi = 0.2$, $p = 0.1$, $\mu = 0.1$ and $\omega = 5$. If we define the measurable scheduling parameter as $\rho(t) \triangleq \sin t$, then the original system can be formulated as a state-delayed LPV system as follows:

$$\dot{x} = \begin{bmatrix} 0 & 1 + c\rho(t) \\ -2 & -3 + p\rho(t) \end{bmatrix} x(t) + \begin{bmatrix} c\rho(t) & 0.1 \\ -0.2 + p\rho(t) & -0.3 \end{bmatrix} x(t - \mu|\cos(\omega t)|) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t),$$

where the parameter space is $[-1 \quad 1]$, $\bar{h} = 0.09$, $\mu = 0.2$ $c = 0.2$, $p = 0.1$ and $\omega = 0$. Note that, due to $\omega = 1$.

results

For the cone complementary linearization algorithm, $k_{\max}$ is chosen to be as $k_{\max} = 20$. Applying the proposed method yields a guaranteed cost $\bar{J} = 2.60$ after $k = 93$ steps. For this example, the other semi-definite programming variables are found as follows:

$$P = \begin{bmatrix} 1.51 & 0.41 \\ 0.41 & 0.35 \end{bmatrix}, \quad X = \begin{bmatrix} 0.04 & -0.10 \\ -0.10 & 0.26 \end{bmatrix}, \tag{3}$$

$$Y = \begin{bmatrix} -0.08 & 0.03 \\ 0.25 & -0.07 \end{bmatrix}, \quad S = \begin{bmatrix} 0.40 & 0 \\ 0 & 0.01 \end{bmatrix}, \tag{4}$$

$$Z = \begin{bmatrix} 0.44 & 0.07 \\ 0.07 & 0.05 \end{bmatrix}, \tag{5}$$

$$K^1 = \begin{bmatrix} -0.04009 - 0.0350 \end{bmatrix}, \quad K^2 = \begin{bmatrix} -0.03977 - 0.03480 \end{bmatrix}, \tag{6}$$

where K^1 and K^2 are the vertex controllers that are associated with the vertices $\theta_1 = -1$ and $\theta_2 = 1$.

Future work

future work

- This method can be re-developed for the systems having norm bounded uncertainties.
- For obtaining less conservative controllers, a new method can also be build up by using parameter dependent Lyapunov functions
- The present problem considered in this work does not comprise the delays in control. Hence the problem can be re-solved by considering both control and state delays.
- The problem does not involve any actuator saturations. Hence, the control problem can be tight by inserting constraints on the control signal.
  ETC.

THANK YOU VERY MUCH FOR YOUR ATTENTION