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Synthesis Inequalities for the Robust PID Controller for a DC Servo Motor and Implementation by Using a Digital Signal Processor

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Abstract

This note considers the ways of obtaining the synthesis equations of the dynamic controller for a DC Servo machine where the closed-loop system poles will move in a Γ-stable region, in spite of the changing system transfer function coefficents by the perturbations. First of all, the mathematical model of a dc servo machine is obtained. Depending upon the varying system transfer function, design equations of the dynamic compensator are found out. These equations includes the ways of choosing the controller coefficents K_P, K_I, K_D which will give a desired response for the tracking system.

1. Introduction

The PID controllers are standard tool for industrial automation. The flexibility of the controller makes it possible to use PID control in many situations. The controllers can be used in cascade control and other controller configurations. Many simple control problems can be handled very well by PID control, provided that the performance requirements are not too high. The basic PID controller algorithm is,

$$u(t) = K_P\left(e(t) + \frac{1}{T_i}\int_0^t e(s)ds + T_d\frac{de}{dt}\right) \qquad (1)$$

where u is the control variable, e is the error defined as $e=\omega_r-\omega$ where ω_r is the reference speed value and ω is the motor output speed.

To get the best performance by the controller algorithm given by the equation (1), it is obvious that we must know the system parameters very well. In the real world, the system parameters vary as the system changes its behaviour by the environmental changes, temperature variations, friction and load. To get the desired response, these variations must be taken under consideration.[7]

In this paper, dynamic controller of a dc servomotor which is 20V and coupled to a load through a coupler is implemented by a digital signal processor. The most important part which makes it different from other controllers is the use of a fast processor which works parallel with a very fast ADC (analogue-digital converter). How it changes the system response by fast conversion of a analogue signal and computation of the control algorithm is examined in this work. It is chosen 8kHz for sampling frequency from the tacho-generator. The controller is designed to make the steady-state error minimum. Compensator coefficients are chosen to satisfy the robustness of the system by detaining the closed-loop system poles in a closed stable Γ-region. Pole cancellation and changing the dynamics of the system by adding new poles and zeros to the system is implemented by the processor. The processor used in this work is, 40Mhz TMS320C50 DSP with 10k x 16 words of on-chip RAM which works parallel with TLC320C40 analogue interface circuitry (AIC) with 14 bit resolution [3]. The processor is communicated with a PC through RS232 serial port. Power Electronics circuit is formed by a drive circuit, power mosfet and opto isolator which separate the low voltage network from the drive circuit. The motor is controlled by a mosfet which works in chopper mode.

The structure of the note is as follows. Section II gives the mathematical model of a dc servo motor. Section III describes the robustness of a feedback system and the Γ-stability. Section IV gives the synthesis equations of the PID controller and discusses the solutions of the synthesis problem of the compansator depending on the limitations on the Γ-stable region. Section V gives the idea of computations of the controller on a digital signal processor.

2. Mathematical model of a dc servo motor

A servomotor is a DC motor designed specifically to be used in a closed-loop control system. The circuit diagram of a servomotor is given in figure 1. In this figure $e_a(t)$ is the armature voltage. R_m is the armature resistance and L_m is the armature inductance. The voltage $e_m(t)$ is the voltage generated in the armature coil because of the motion of the coil in the motor's magnetic field and is usually called back –EMF [4].

Then;

$$e_m(t) = K\Phi \frac{d\theta}{dt} \tag{2}$$

where K is the motor parameter, Φ is the field flux and θ is the angle of the motor shaft. We assume that the flux does not change during the operation. Hence

$$e_m(t) = K_m \frac{d\theta}{dt} \quad . \tag{3}$$

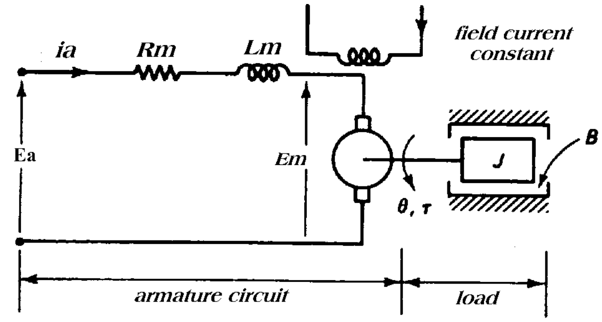


Figure 1 Circuit diagram of a servomotor

The Laplace transform of (3) gives

$$E_m(s)=K_m s\theta(s) \quad . \tag{4}$$

For the armature circuit we can write

$$E_a(s)=(L_m s+R_m).I_a(s)+E_m(s) \, . \tag{5}$$

The mathematical equation for the developed torque is

$$\tau(t)=K_1\Phi I_a(t)=K_\tau i_a(t) \, . \tag{6}$$

Finally, for the mechanical terminal pair we can write,

$$J\frac{d^2\theta}{dt^2} = \tau(t) - \beta\frac{d\theta}{dt} \tag{7}$$

where, J is the inertia connected to the motor shaft and β includes the friction coefficents. Solving these set of equations for the motor shaft angle yields [4]

$$\theta(s) = \frac{T(s)}{Js^2 + \beta s} \quad . \tag{8}$$

as a result of these equations, one can obtain the motor transfer function

$$G(s)=\frac{w(s)}{E_a(s)}=\frac{K_\tau}{JR_m s+(\beta R_m+K_\tau K_m)} \qquad (9)$$

3. Γ - stability and robustness analysis

In a Γ-region which is enclosed by a border of δΓ can be expressed by, $\delta\Gamma \Leftarrow \{s \mid s=\sigma(\alpha)+jw(\alpha) \quad \alpha\in[\alpha^-, \alpha^+]\}$. α^- and α^+ borders can be -∞ or +∞. α parameter is called as the generalized frequency (figure 2).

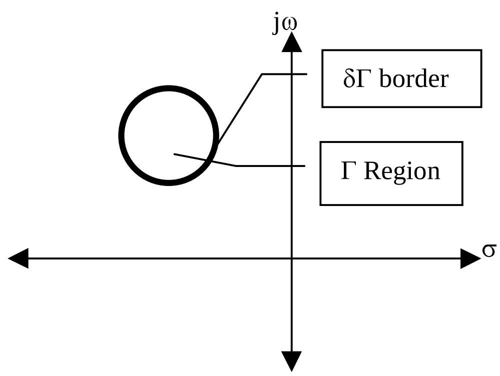


Figure 2 : Γ-Stable region and the *δΓ*-stability border

Our desire is to place our closed loop poles in a Γ-region which is on the left and side of the segment as shown in figure 3.

Poles of the feedback control system will be detained by the segment whose mathematical expression can be defined by

$$\sigma=\xi\omega_n \qquad (10)$$

and if we solve equation (3.1) for *ω*,

$$\omega^2=\omega_n{}^2(1-\xi^2) \quad . \qquad (11)$$

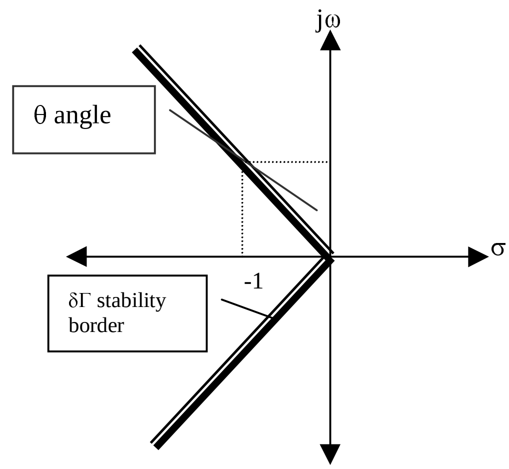


Figure 3 : Desired Closed loop pole location

It is obvious that,

$$\omega=\frac{\sigma}{\xi}\sqrt{1-\xi^2} \qquad (12)$$

and the border equation can be expressed as,

$$s=\alpha+j\omega=\sigma+j\frac{\sigma\sqrt{1-\xi^2}}{\xi} \quad (\sigma\leq 0)\ . \qquad (13)$$

Because of the limitations of ζ and

$$\zeta=\sin\theta=\frac{1}{\sqrt{2}} \quad , \qquad (14)$$

ζ can vary up to equation (3.5). If we choose the generalized frequency as α ≡ σ and express the border by the generalized frequency then we obtain

$$s=\alpha+j\frac{\alpha\sqrt{1-\xi^2}}{\xi} \quad (\alpha\leq -a)\ . \qquad (15)$$

Specially for the value of $\xi = \frac{1}{\sqrt{2}}$ the border equation will be,

$$S(\alpha) = \sigma + j\sigma = \alpha + j\alpha \qquad (16)$$

3.1 Ackermann, Koesbaurr theorem

If we express the characteristic polynomial by $\Delta(s,\vec{q}) = \begin{bmatrix} 1 & s & . & . & s^n \end{bmatrix} \vec{a}(\vec{q})$ then the roots of the polynomial for the border will provide [5]

$Q_{m}(\alpha) \hat{=} \{\vec{q} \mid \quad \Delta(\sigma(\alpha) + jw(\alpha), \vec{q}) = 0 \}$. (17)

For $\alpha \in [\alpha^-, \alpha^+]$ and $\vec{q} \in \theta_{ým}(\alpha)$ it must provide the following equation,

$$\begin{bmatrix} d_0(\alpha) & d_1(\alpha) & . & . & d_n(\alpha) \\ 0 & d_0(\alpha) & . & . & d_{n-1}(\alpha) \end{bmatrix} \vec{\theta}(\vec{q}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (18)$$

where

$$d_0(\alpha) = 1$$
$$d_1(\alpha) = 2\sigma(\alpha)$$
.
.
$$d_{i+1} = 2\sigma(\alpha) d_i(\alpha) - (\sigma^2(\alpha) + w^2(\alpha)) d_{i-1}(\alpha)$$
$$(i = 1,2,..,n-1) \ . \qquad (19)$$

The proof of the equations expressed in (18) and (19), are well explained by Ackermann in [5].

4. Sythesis Inequalities for the controller

The characteristic equation of the automatic control system which is given by the block diagram in figure will be,

$$s^3 + \frac{\beta R_m T + K_\tau K_m T + K_p K_\tau T + J R_m}{J R_m T} s^2 + \frac{\beta R_m + K_\tau K_m + K_p K_\tau + K_D K_\tau}{J R_m T} s + \frac{K_I K_\tau}{J R_m T} = 0 \qquad (20)$$

In generalized form (20) can be expressed as $s^3 + a_2 s^2 + a_1 s + a_0 = 0$ and if the motor parameters vary in a given interval as $\beta \in (\beta_-, \beta_+)$, K_r ≅ *constant* , R_m≅ *constant* , K_p = *constant* , K_I=*constant* and finally $J \in (J_-, J_+)$. Then the coefficents of the characteristic polynomial will move between a region defined by, $a_2^- \leq a_2 \leq a_2^+$ and $a_1^- \leq a_1 \leq a_1^+$ respectively. If we choose K_I such that it will satisfy,

$\frac{K_I K_\tau}{J R_m T} = 1$ then K_I will be determined by,

$$K_I = \frac{J R_m T}{K_\tau} \qquad (21)$$

If we use the equations (18) and (19) respectively then we will get the following relations between a_1, a_2 and α.

$$a_1 = \frac{2\alpha^3 - 1}{\alpha} = 2\alpha^2 - \frac{1}{\alpha} \qquad (22)$$

$$a_2 = \frac{1}{2\alpha^2} - 2\alpha \qquad (23)$$

The plot of the given equations of (22) and (23) will be as shown in figure 4. To implement the nonlinear relations by a signal processor, a convex region must be chosen on the plot. We will try to push the coefficents of the transfer function into the convex region shown in figure 4.

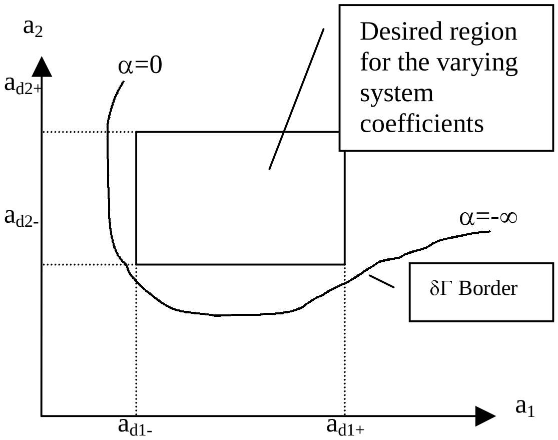


Figure 4: Desired Γ-Stable region for the closed loop transfer function coefficents

To do this, we will use the compensator coefficents, then we will get the following limitations on choosing the compensator coefficients. If we use the worst coefficients in the given transfer function (20); we should choose the compensator coefficients as shown by (4.5) and (4.6) respectively.

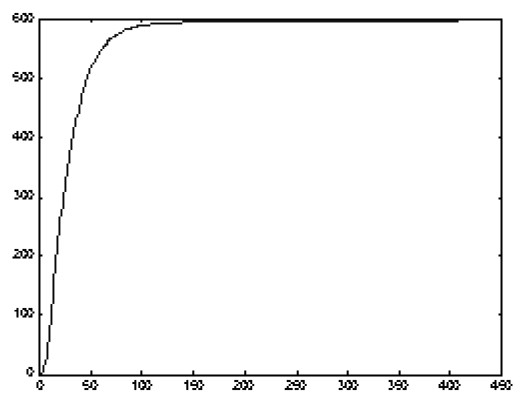
$$\frac{j^{+}R_{m}Ta_{2d-}-\beta^{-}R_{m}T-K_{\tau}K_{m}T-J^{-}R_{m}}{K_{\tau}T}\leq K_{p}\leq\frac{a_{2d+}J^{-}R_{m}T-\beta^{+}R_{m}T-K_{\tau}K_{m}T-J^{+}R_{m}}{K_{\tau}T} \tag{24}$$

$$\frac{a_{1d-}J^{+}R_{m}T-\beta^{-}R_{m}-K_{\tau}K_{m}-K_{p}{}^{-}K_{\tau}}{K_{\tau}}\leq K_{D}\leq\frac{a_{1d+}J^{-}R_{m}T-\beta^{+}R_{m}-K_{\tau}K_{m}-K_{p}{}^{+}K_{\tau}}{K_{\tau}} \tag{25}$$

5. Conclusion

During the implementation process; the controller coefficents are computed and the compensator in Laplace domain is found out, then by Z-Transform the compensator equivalent is computed in digital domain and programmed by DSP. As a result it has seen that the controller forces the motor to move as the reference speed and the controller does not miss the domination on the motor as the motor coefficients vary by the disturbances. Reference speed is obtained by software and the output speed of the system is measured by a digital tachometer.

Figure 5: System response to the reference speed



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