

# Direct Adaptive Fuzzy Logic Controller with Self-Tuning Input Scaling Factors

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## Abstract

It is well known that in a fuzzy logic controller real problem lies on designing the rule base and choosing the input and output scaling factors. In this paper direct adaptive fuzzy logic controller with self-tuning scaling factors is proposed. The adaptive mechanism tunes the rule base of the fuzzy controller in order to improve the performance whereas another fuzzy controller adjust the input scaling factors for a better control performance. In order to demonstrate the validity of the proposed controller, the algorithm is applied to a servo-system which includes a PMDC motor and a non-linear load. Experimental results show that a remarkable performance is achieved when it is compared with a conventional fuzzy –PI controller.

## 1 Introduction

Fuzzy Logic Control (FLC) was first introduced and applied to control problems in 1970's in an attempt to design controllers for systems that are structurally difficult to model. [1], [2], [3]. Since then the field incredibly broadened. Many successful applications and significant research works followed one after the other. Some of the milestones are water quality control application with fuzzy logic [4], automatic train operating systems [5], elevator control [6], reactor control [7]. Detailed information can be found in [8].

However, at present there is no systematic procedure for the design of an FLC when it is compared with non-fuzzy counterparts like linear and non-linear control systems. The difficulty lies under the highly nonlinear structure of the FLC. Many research works have been presented to overcome this problem. Existing the research work presented in literature on the design procedure of a FLC can be divided into 3 categories. These are: the researches that are related with the design procedures based on the scaling factor adjustment, [3], [9], [10], [11] the researches that are related with the design procedures based on the rule base adjustment [11], [12], [13] and the research works that are related with the design procedures based on the membership function adjustment [14], [15], respectively. This research work can be categorized as partially a study on scaling factor and a study on membership tuning.

Fuzzy controllers are supposed to work in situations where there is a large uncertainty or unknown variation in plant parameters and structures. Generally the basic objective of adaptive control is to maintain consistent performance of a system in the presence of these uncertainties. Therefore advanced fuzzy control mechanism should include adaptive characteristics. Depending upon the human knowledge used and the structure of the fuzzy controller used, adaptive fuzzy control is classified into three main categories. These are indirect adaptive fuzzy control where the fuzzy controller comprises some plant knowledge; direct adaptive fuzzy control where the controller comprises some control knowledge and combined direct/indirect adaptive fuzzy control where the controller comprises both some plant knowledge and some control knowledge [16]. This study can be put into both the category of direct adaptive fuzzy control and the category of scaling factor adjustment since the adaptive mechanism is responsible of tuning the indexes of the output membership functions, whereas another rule based system is responsible of tuning the input scaling factors. To demonstrate the validity and the effectiveness of the proposed controller we have applied the controller to a PMDC servomotor system (52V, 2.2A, 92W, 3000rpm.), which is coupled to a non-linear load. All the algorithms proposed in the study are programmed under C++ and applied to the system via Pentium® PC. Fig. 1 shows the block diagram of the proposed controller.

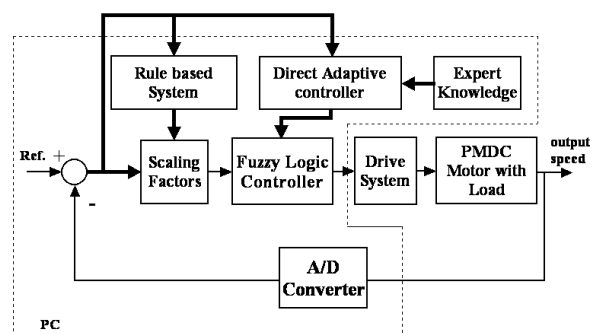


Figure 1: Block diagram of the proposed control system

The rest of the paper is organized as follows. Section 2 reviews the direct adaptive fuzzy control algorithm, Section 3 presents the details of rule-based input scaling factor adjustment mechanism, Section 4 presents the

experimental results of the proposed control system with discussions and finally Section 5 concludes the paper.

## 2 Direct Adaptive Fuzzy Control

Any linear or nonlinear plant can be represented as

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \dots, x^{(n-1)}) + bu \\ y &= x \end{aligned} \quad (1)$$

where  $f$  is an unknown function and  $b$  is an unknown positive constant.  $u \in \mathfrak{X}$  and  $y \in \mathfrak{Y}$  are the input and output of the plant, respectively, and  $\vec{x} = (x_1, x_2, \dots, x_n)^T = (x, \dot{x}, \dots, x^{(n-1)})^T \in \mathfrak{X}^n$  is the state vector of the system that is assumed to be measurable for the system. In order for (1) to be controllable, we assume that  $b > 0$ . The control objective is to design a feedback controller  $u = u(\vec{x} | \vec{\theta})$  for adjusting the parameter vector  $\vec{\theta}$ , such that the output  $y$  follows the reference  $y_m$  as close as possible. Here we are provided to have some control knowledge, in terms of some fuzzy rules like:

$$\text{IF } x_1 \text{ is } P_1^r \text{ and } \dots \text{ and } x_n \text{ is } P_n^r, \text{ THEN } u \text{ is } Q^r, \quad (2)$$

where  $P_i^r$  and  $Q^r$  are the fuzzy sets in  $\mathfrak{X}$ , and  $r=1, 2, \dots, L_u$ . For every state  $x_i$ , one can define ( $i=1, 2, \dots, n$ )  $m_i$  fuzzy sets  $A_i^{l_i}$  ( $l_i=1, 2, \dots, m_i$ ), which include  $P_i^r$  ( $r=1, 2, \dots, L_u$ ) in (2) as special cases. As a second step, the fuzzy rule based is construct by  $\prod_{i=1}^n m_i$  rules such as,

$$\text{IF } x_1 \text{ is } A_1^{l_1} \text{ and } \dots \text{ and } x_n \text{ is } A_n^{l_n}, \text{ THEN } u \text{ is } S^{l_1 \dots l_n}, \quad (3)$$

where  $l_i=1, 2, \dots, m_i, i=1, 2, \dots, n$  and  $S^{l_1 \dots l_n}$  is same with  $Q^r$ , if the IF parts of (3) are same with the IF parts of (2) and equals to some arbitrary fuzzy set otherwise. By using product inference engine, singleton fuzzifier and center of average defuzzifier,

$$u(\vec{x} | \vec{\theta}) = \frac{\sum_{l_1=1}^{m_1} \dots \sum_{l_n=1}^{m_n} \bar{y}_u^{l_1 \dots l_n} \left[ \prod_{i=1}^n \mu_{A_i^{l_i}}(x_i) \right]}{\sum_{l_1=1}^{m_1} \dots \sum_{l_n=1}^{m_n} \left[ \prod_{i=1}^n \mu_{A_i^{l_i}}(x_i) \right]}, \quad (4)$$

is obtained, where each  $\bar{y}_u^{l_1 \dots l_n}$  can be defined as adjustable index of the output space of the fuzzy controller for a rule.

By collecting the adjustable parameters into  $\vec{\theta}$ , (4) can be rewritten as

$$u(\vec{x}) = \vec{\theta}^T \vec{\xi}(\vec{x}), \quad (5)$$

where

$$\vec{\xi}(\vec{x}) = \frac{\prod_{i=1}^n \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{m_1} \dots \sum_{l_n=1}^{m_n} \left[ \prod_{i=1}^n \mu_{A_i^{l_i}}(x_i) \right]}. \quad (6)$$

Let  $e = y_m - y$ ,  $\vec{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$ ,  $\vec{k} = (k_n, \dots, k_1)^T$ , such that all the roots of the polynomial  $s^n + k_1 s^{n-1} + \dots + k_n$  are in the open left-half complex plane and  $u^*$  be the ideal control law described as

$$u^* = \frac{1}{b} \left( -f(\vec{x}) + y_m^{(n)} + \vec{k}^T \vec{e} \right). \quad (7)$$

Substituting (7) in (1) and rearranging the terms to result an error dynamics, yields:

$$e^{(n)} = -\vec{k}^T \vec{e} + b(u^* - u(\vec{x} | \vec{\theta})). \quad (8)$$

Letting ,

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ -k_n & -k_{n-1} & \dots & \dots & \dots & \dots & -k_1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ \dots \\ 0 \\ b \end{bmatrix}, \quad (9)$$

the closed-loop error dynamics can be rewritten as,

$$\dot{\vec{e}} = \Lambda \vec{e} + \vec{b}(u^* - u(\vec{x} | \vec{\theta})). \quad (10)$$

Let the optimal adjustable parameters vector be as,

$$\vec{\theta}^* = \arg \min_{\vec{\theta} \in \mathfrak{X}} \left( \sup_{\vec{x} \in \mathfrak{X}^n} |u(\vec{x} | \vec{\theta}) - u^*| \right). \quad (11)$$

Also the minimum approximation error can be defined as,

$$w = u(\vec{x} | \vec{\theta}^*) - u^*. \quad (12)$$

By using (12), (10) can be rewritten as,

$$\dot{\vec{e}} = \Lambda \vec{e} + \vec{b}(\vec{\theta}^* - \vec{\theta})^T \vec{\xi}(\vec{x}) - \vec{b}w. \quad (13)$$

Let the Lyapunov energy function be like,

$$V = \frac{1}{2} \bar{e}^T \mathbf{P} \bar{e} + \frac{b}{2\gamma} (\bar{\theta}^* - \bar{\theta})^T (\bar{\theta}^* - \bar{\theta}). \quad (14)$$

Where  $\mathbf{P}$  is positive definite matrix satisfying the Lyapunov equation (15), and  $b > 0$ ,  $\gamma > 0$ .

$$\Lambda^T \mathbf{P} + \mathbf{P} \Lambda = -\mathbf{Q}. \quad (15)$$

The time derivative of  $V$  is,

$$\dot{V} = -\frac{1}{2} \bar{e}^T \mathbf{Q} \bar{e} + \bar{e}^T \mathbf{P} \bar{b} [(\bar{\theta}^* - \bar{\theta})^T \bar{\xi}(\bar{x}) - w] - \frac{b}{\gamma} (\bar{\theta}^* - \bar{\theta})^T \dot{\bar{\theta}}. \quad (16)$$

If we denote the last column of  $\mathbf{P}$  as  $\bar{p}_n$  then one can rewrite (16) as,

$$\dot{V} = -\frac{1}{2} \bar{e}^T \mathbf{Q} \bar{e} + \frac{b}{\gamma} (\bar{\theta}^* - \bar{\theta})^T [\gamma \bar{e}^T \bar{p}_n \bar{\xi}(\bar{x}) - \dot{\bar{\theta}}] - \bar{e}^T \bar{p}_n b w. \quad (17)$$

Then the adaptation law that is chosen as (18) will satisfy  $\dot{V} < 0$  under the condition: The rule base should have sufficiently large number of rules [16], [17].

$$\dot{\bar{\theta}} = \gamma \bar{e}^T \bar{p}_n \bar{\xi}(\bar{x}) \quad (18)$$

### 3 Rule-Based Scaling Factor Adjustment Mechanism

It is well known that PI (Proportional-Integral) type fuzzy logic controllers show better performance in steady state than PD (Proportional – Derivative) type fuzzy controllers. However PD type fuzzy controllers are faster at transient state than PI type fuzzy controllers. In a fuzzy controller the scaling factors ( $G_e$ ,  $G_{ce}$ ) are very dominant on the performance of the controller. In order to improve the transient performance of PI type fuzzy controllers we propose a rule-based scaling factor adjustment mechanism (SFAM). The strategy is that the scaling factors of the fuzzy controller should be increased at the beginning when the error is big, and should be reduced when the error is small for avoiding the undesirable overshoots. The absolute value of normalized error signal  $e$  (19) is used as input to the scaling factor adjustment mechanism.

$$R = |e / (y_m - y_{initial})| \quad (19)$$

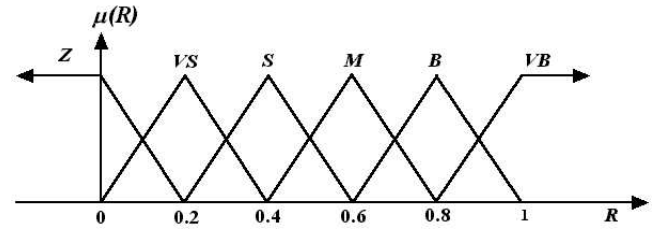
By using the above reasoning, the rule base for scaling factor adjustment mechanism is constructed as shown in Table 1, where  $\Delta G_e$  represents the output of SFAM for  $G_e$  and  $\Delta G_{ce}$  represents the output of SFAM for  $G_{ce}$ . Here Z =Zero, VS = Very Small, S= Small, M= Medium, B = Big,

VB=Very Big, NS = Negative Small, NM= Negative Medium, PS = Positive Small, PM = Positive Medium and PB = Positive Big, respectively.

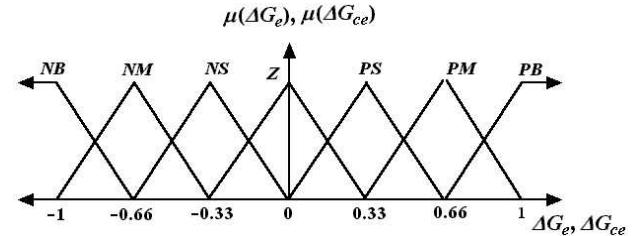
**Table 1:** Rule Base for SFAM

R	Z	VS	S	M	B	VB
$\Delta G_e$	Z	NS	NM	PS	PM	PB
$\Delta G_{ce}$	Z	NS	NM	PS	PM	PB

The input and output membership functions of SFAM are shown in Fig. 2 and Fig. 3, respectively. In order to reduce the computational complexity, all the membership functions are chosen to be triangular form.



**Figure 2:** Input membership functions of SFAM



**Figure 3:** Output membership functions of SFAM

Then at every sampling instant, the input scaling factors are updated as follows

$$\begin{aligned} G_e &= G_{eN} + \Delta G_e / (|\text{Set point} - \text{initial output}|) \\ G_{ce} &= G_{ceN} + \Delta G_{ce} / (|\text{Set point} - \text{initial output}|) \end{aligned}, \quad (20)$$

where  $G_{eN}$ , and  $G_{ceN}$  represents the nominal values of  $G_e$  and  $G_{ce}$  respectively.

These nominal values are determined by trial and error method during the design process of the controller. Generally these values are chosen to map the maximum positive error value to 1, and maximum negative error value to -1.

## 4 Experimental Results

In this section, we show the real time performance results of the proposed adaptive fuzzy PI controller. The controller performances are compared on a permanent magnet DC servo motor mechanism with a nonlinear load increasing with the speed of the system. The amount of the load increases exponentially with the increasing speed of the rotor. The fuzzy controller adjusts the speed of the PMDC motor by changing the supply voltage via a chopper circuit.

The sampling period  $T_s$  is chosen as 2kHz. All the software is coded with C++ and applied to the system via Pentium 200Mhz PC. The drive circuit is shown in Fig.10. The process has three DC motors, which are 90W each, and are coupled to each other on the same rotor axis. One of them is used for feedback generation and the others are used as plant and load, respectively. The feedback signal is eliminated from disturbances by a passive filter and a digital moving average type 5<sup>th</sup> order digital filter. The characteristics of the controlled motor are listed in Table 4.

In general, speed control of a PMDC motor system can be modeled by a second order differential equation as,

$$\frac{\omega(s)}{U(s)} = \frac{\lambda K / JL}{s^2 + \frac{\beta L + JR}{JL} s + \frac{\beta R + K^2}{JL}}, \quad (21)$$

where  $\lambda$  denotes the controller gain,  $K$  denotes the motor constant,  $J$  denotes total inertia of motor and load,  $L$  denotes armature inductance,  $R$  denotes armature resistance,  $\beta$  denotes the total friction coefficients of load and motor,  $\omega$  denotes the angular velocity of rotor and finally  $U$  denotes the applied voltage to motor.

The rule base for the main fuzzy controller is chosen to be like in Table 3. This kind of rule base is very general for PI type fuzzy controllers with 2 inputs and robust enough for a wide range of applications. It is designed by phase-plane technique. The membership functions for the speed error and for the change in the speed error are chosen as shown in Fig. 4 and Fig. 5, respectively. The defuzzification membership functions are chosen as in Fig. 6 where all the sub fuzzy sets are distributed equally to cover the output space. The index values of these output fuzzy sets forms the initial values. All new values of these output indexes are updated by (18) during the run-time of the system.

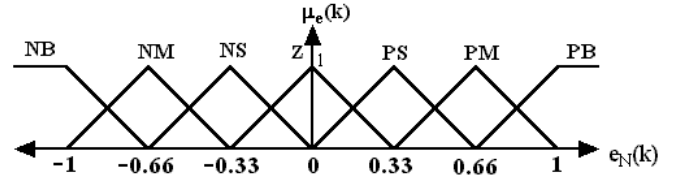
For this study we chose the parameters as follows:  $n = 2$ ,  $L_u = 7$ ,  $m_1 = 7$ ,  $m_2 = 7$ ,  $k_1 = 1$ ,  $k_2 = 1$ ,  $\mathbf{Q} = \text{diag}(0.1, 0.1)$ . Then (4) takes the form as,

$$\Delta u(\vec{x} | \vec{\theta}) = \frac{\sum_{i_1=1}^7 \sum_{i_2=1}^7 \bar{y}_u^{i_1 i_2} \left[ \prod_{i=1}^2 \mu_{A_i^{i_1}}(x_i) \right]}{\sum_{i_1=1}^7 \sum_{i_2=1}^7 \left[ \prod_{i=1}^2 \mu_{A_i^{i_1}}(x_i) \right]}, \quad (22)$$

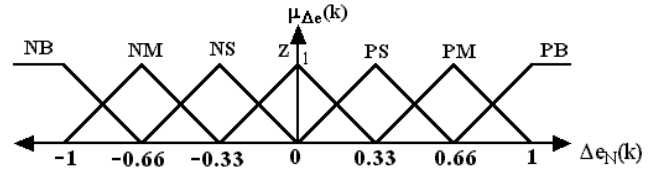
since  $u(k) = u(k-1) + \Delta u(\vec{x} | \vec{\theta})$ .

**Table 2:** Rule base for the main FLC

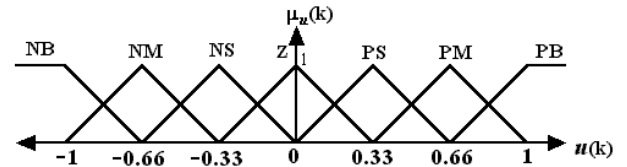
		$e$						
		NB	NM	NS	Z	PS	PM	PB
$\Delta e$	NB	NB	NB	NB	NB	NM	NS	Z
	NM	NB	NB	NB	NM	NS	Z	PS
	NS	NB	NB	NM	NS	Z	PS	PM
	Z	NB	NM	NS	Z	PS	PM	PB
	PS	NM	NS	Z	PS	PM	PB	PB
	PM	NS	Z	PS	PM	PB	PB	PB
	PB	Z	PS	PM	PB	PB	PB	PB



**Figure 4:** Fuzzification functions for  $e$



**Figure 5:** Fuzzification functions for  $\Delta e$



**Figure 6** Defuzzification functions for  $u$

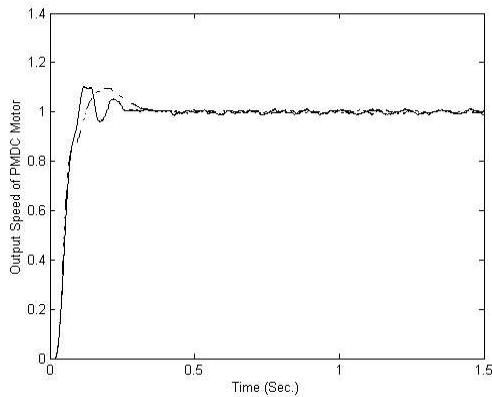
Fig. 7 shows the step response of the proposed control algorithm together with the conventional fuzzy PI controller response. Fig. 8 shows the variation of  $G_e$  and  $G_{ce}$  during run-time, whereas Fig 9 shows the variation of index values of output fuzzy sets of adaptive fuzzy-PI controller. Experimental results states that a remarkable performance improvement achieved when it is compared with its non-adaptive conventional counterpart. For comparison, Table 3 summarizes the performance results of the proposed controller together with the conventional fuzzy PI controller. Here two integral criteria IAE and ITAE are considered, since visual observations are not always enough to make good comparison between different types of controllers. Large errors such as errors occurred in transient regime contribute heavily to IAE (Integral of

Absolute Error), whereas ITAE (Integral of Time Weighted Absolute Error) penalizes heavily errors that occur in steady state regime. The following are the equations that correspond to IAE and ITAE at time instant  $k$ , respectively.

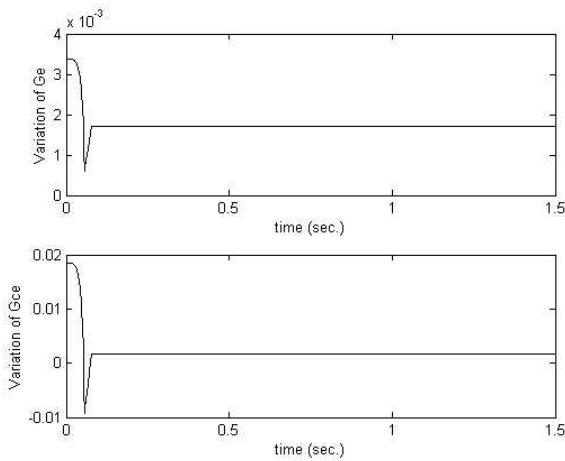
$$IAE(k) = \sum_{i=0}^k T_s e(k) \quad k = 0, 1, \dots, n$$

$$ITAE(k) = \sum_{i=0}^k k T_s e(k) \quad k = 0, 1, \dots, n \quad (23)$$

Here,  $T_s$  symbolizes the sampling period.



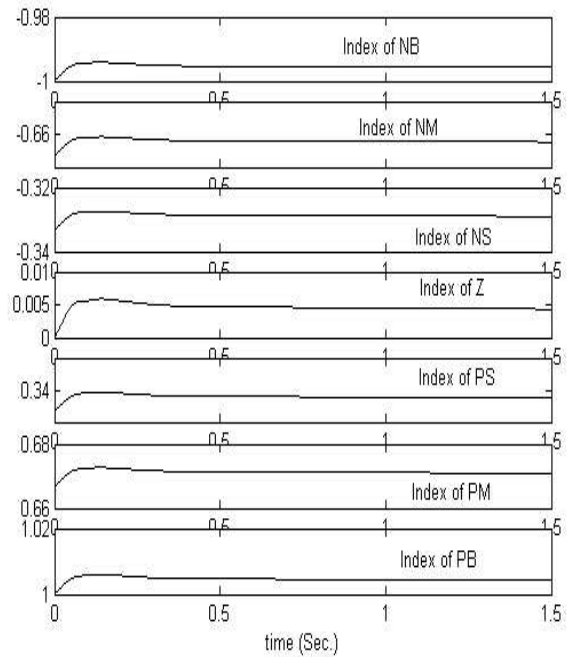
**Figure 7:** Step response of the proposed controller (—) versus Conventional Fuzzy PI (---)



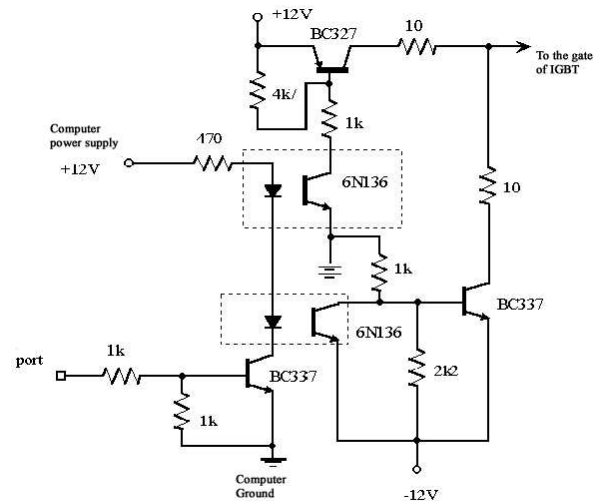
**Figure 8:** Variations of  $G_e$  and  $G_{ce}$

**Table 3:** Performance analysis of the controllers

	IAE	ITAE	$t_r$ (sec.)	%OS	$t_s$ (sec.)
<b>Proposed Cont.</b>	82165	9004	0.1	10.2	0.24
<b>Conv. Fuzzy PI</b>	91146	9568	0.125	10	0.33



**Figure 9:** Variations of index values of output fuzzy sets for the proposed fuzzy controller



**Figure 10:** Drive circuit of the PMDC motor system

## 5. Conclusion

We proposed a new FLC architecture where the input scaling factors are adapted by an independent fuzzy adaptation mechanism. On the other hand the proposed fuzzy controller is powered by an on-line adaptive mechanism, which tunes the index values of the output fuzzy sets. In order to demonstrate the validity of the proposed controller, the algorithm is applied to a servo-system which includes a PMDC motor and a non-linear load. Experimental results show that a remarkable performance is achieved when it is compared

with a conventional fuzzy –PI controller.

**Table 4:** Parameters of PMDC Motor used in experiment

$U_{nominal}$	52V
P	92W
$I_{nominal}$	2.2A
$n_{nominal}$	3000rpm.
L	0.0015H
K	0.14Vsec.rad <sup>-1</sup>
$\beta$	0.00011NmSecRad <sup>-1</sup>
J	0.0000421kgm <sup>2</sup>
R	2.9 ohm

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