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Omurlu, Vasfi Emre; Engin, Seref Naci; Kucukdemiral, I. B.

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A Robust Single Input Adaptive Sliding Mode Fuzzy Logic Controller for a Nonlinear Automotive Suspension System

V.E. Ömürlü
Yildiz Technical University, Mechanical Engineering Dept., Istanbul, Turkey
Email omurlu.1@osu.edu

Ş.N. Engin,
Yildiz Technical University, Electrical Engineering Dept., Istanbul, Turkey
Email: nengin@yildiz.edu.tr

İ.B. Küçükdemiral
Yildiz Technical University, Electrical Engineering Dept., Istanbul, Turkey
Email:beklan@yildiz.edu.tr

Abstract

A variety of control architectures have been investigated for active suspension systems to improve ride comfort and holding ability of passenger cars. The most popular ones among these are the linear quadratic and fuzzy logic based controllers. While the former cannot produce desired outputs when there are strong nonlinearities and non-negligible disturbances, the latter overcomes such problems although it requires a comprehensive rule-base involving heavy computation loads. The proposed controller in this paper, which unifies the capability of fuzzy logic with the robustness of sliding mode controller, presents prevailing results and hence surpasses them with its adaptive architecture and proves to overcome the global stability problem. Effectiveness of the controller and the performance comparison with chosen control techniques including PID and PD type self-tuning fuzzy controller is performed on a 2DOF quarter car model which consists of component-wise nonlinearities.

1. Introduction

Research works on active suspension systems for automotive industry have drawn a great deal of interest in recent years both from academia and automobile industry, [1]. There are two major objectives in these studies that are to improve ride comfort by reducing the vertical acceleration of the sprung mass and to increase holding ability of the vehicle by providing adequate suspension deflections. The research works on control of Automotive Active Suspension Systems (AASSs) are mostly based on the Linear Quadratic (LQ) control theory, [2-5]. However, even a simple car model is a multivariable, relatively complicated system and involves nonlinear sub-systems and non-negligible disturbances. Therefore the controller that is based on the LQ theory may not be reliable when perturbed conditions occurred.

To overcome the problems that originate from the complexity and non-linearity of vehicle systems such as the one used in this paper, various kinds of Fuzzy Logic Controllers (FLCs) are suggested [6-7]. The prominent superiority of the FLC is that, it can effectively control complex, ill-defined systems involving nonlinearities, parameter variations and disturbances just like the suspension systems in vehicles. The control method, which models the way of human thinking and decision-making can have many advantages, such as using past experiences, making generalizations, being robust and involving only simple calculations rather than necessitating exact mathematical descriptions of the system to be controlled. This is particularly advantageous in controlling nonlinear plants. Successful applications have been reported for a number of complex and non-linear processes, [8-10]. The majority of the research works reported in the field of fuzzy control deal with 2-input, 1-output structure, where the inputs are generally the error and the change rate of the error, and the output is either the control signal or the change rate of it. Such fuzzy logic controllers are considerably suitable for simple plants, where low-order plant models are dealt with. The higher order models represent relatively more complex plants, which can exhibit various dynamic behavior states. Designing an FLC for such plants requires a multi-dimensional rule base involving hundreds of computations. Moreover, the construction of such a rule-base will be a cumbersome and tedious job even for a specialist of the plant of interest. Finally the most important problem to be solved is the global stability of the closed-loop system which was addressed in [11].

Taking these drawbacks into consideration, we propose a novel robust, simple and industrially applicable FLC with a single state feedback for AASSs, where the stability of the controller is proven in the sense of Lyapunov stability. The other benefit is that the rule-base for the proposed controller does not need to be tuned since an adaptation mechanism takes the responsibility for tuning.

2. Modeling the nonlinear active suspension system

A typical active suspension system for a quarter car model is illustrated in Figure 1. The wheel is connected to the car body through a massless axle. The tire is modeled as a simple linear spring attached between the wheel and the ground. It is assumed that the tire never leaves the ground. The motion of the axle of the suspension system is controlled by an external actuator force, damper and spring combination. The actuator force is denoted with f_u. Nonlinear damping and spring forces are provided as

$$f_s = k_s(z_u - z_s) + \left(k_s/4\right)\left(z_u^{\,3} - z_s^{\,3}\right)$$

and

$$f_b = b_s \cdot \left|\dot{z}_u - \dot{z}_s\right| \cdot \left(\dot{z}_u - \dot{z}_s\right)$$

in which damping coefficient and spring coefficient are denoted with, b_s and, k_s respectively [12]. Car body displacement, z_s wheel displacement z_u and road displacement z_r are all measured from the static equilibrium position.

The dynamic equations of the quarter car active suspension system are,

$$\begin{aligned} m_s\ddot{z}_s &= f_s + f_b + f_u \\ m_u\ddot{z}_u &= -f_s - f_b - f_u + k_t(z_r - z_u) \end{aligned} \tag{1}$$

where m_s and m_u are the masses of the car body and the wheel respectively.

3. Design of Robust Single-Input Adaptive Fuzzy Sliding Mode Controller for Nonlinear AASS

In general, compact dynamic equation for AASS can be regarded as a second order differential equation, such as

$$M\ddot{z}_s(t) + B\dot{z}_s(t) + K\dot{z}_s(t) + T_d = F_u \tag{2}$$

where M is the total mass of the body, B is the damping coefficient, T_d is the total definition for unknown load disturbances such as noise or friction terms that effect the body and finally F_u is the applied control force to the system. By rearranging terms, (2) can be rewritten in the following form:

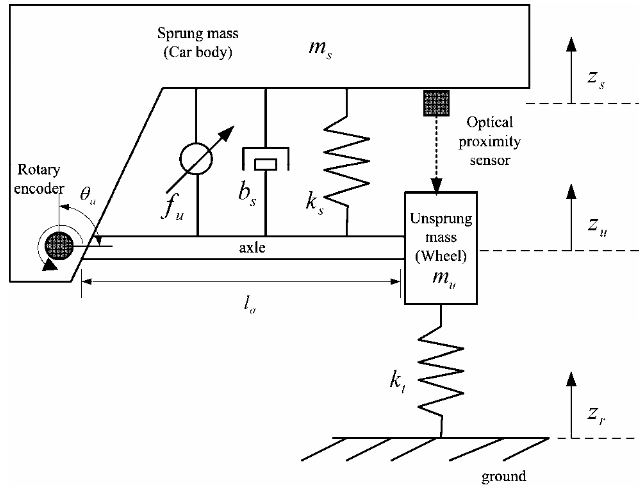

Figure 1. Nonlinear quarter car model used

$$\begin{aligned} \ddot{z}_s(t) &= -\frac{B}{M}\dot{z}_s(t) - \frac{K}{M}z_s(t) - \frac{T_d}{M} + \frac{F_u}{M} \\ &\equiv A_p\dot{z}_s(t) + K_p z_s(t) + D_p T_d + M_p u(t) \end{aligned} \tag{3}$$

where A_p=-B/M, K_p=-K/M, D_p=-$1/M$, M_p=$1/M$ and $u(t)$ is the control effort. The control objective is to find a suitable control signal so that the body acceleration of the sprung mass is minimized. The error of sprung mass displacement is

$$e(t) = z_s(t) - z_d(t) \tag{4}$$

where $z_d(t)$ is the desired displacement of the car body. The first step of sliding mode controller design process is choosing the sliding surface which is directly related to stability and the time response of the closed loop system. The second step of the sliding mode controller design process consists of determination process of the control signal such that the state trajectories are forced towards the sliding switching surface. Although linear switching surfaces are addressed in many articles, a nonlinear proportional-integrator type surface is used in the proposed design as in [13], since the conventional linear sliding surfaces are insufficient to eliminate the static steady-state errors and produces sharp control signals. Therefore, following sliding surface is employed,

$$s(t) = \dot{z}_s(t) - \int_0^t \left[\ddot{z}_d(\tau) - k_1\dot{e}(\tau) - k_2 e(\tau)\right] d\tau \tag{5}$$

This produces smooth control signals and is capable of eliminating the steady-state errors. k_1 and k_2 are both non-zero positive constants and if the state trajectory of the system is trapped on the sliding surface $s(t) = \dot{s}(t)$ then the equivalent dynamics of the AASS will be,

$$\ddot{e}(t) + k_1\dot{e}(t) + k_2 e(t) = 0 \tag{6}$$

It is obvious that the error dynamics in (6) is exponentially stable if-and-only-if (6) is Hurwitz. Assuming that the system dynamics (2) is well known, a feedback linearization such as,

$$u^*(t) = M_p^{-1}\begin{bmatrix} -A_p \dot{z}_s(t) - K_p z_s(t) - D_p T_d \\ +\ddot{z}_d(t) - k_1 \dot{e}(t) - k_2 e(t) \end{bmatrix} \quad (7)$$

achieves (6) when the control law (7) is applied to (3). However, generally, the system dynamics and disturbances acting on the system cannot exactly be known. Therefore, control law defined by (7) cannot be directly applicable. Thus, one method to overcome this difficulty is to imitate the feedback linearization method given in (7) by an adaptive fuzzy logic controller such as

$$u_{\mathrm{fz}}(s) = \frac{\sum_{r=1}^{N} w_r \theta_r}{\sum_{r=1}^{N} w_r} \quad (8)$$

where θ_r, $r=1,2,\ldots,N$ are the discrete singleton control signals labeled as adjustable parameters and w_r is the firing weight of the rth rule. On the other hand the fuzzy rules are in following form:

Rule r : IF s is Ω_s^r , THEN u is θ_r (9)

If θ_r is chosen as an adjustable parameter, (8) can be rewritten as

$$u_{\mathrm{fz}} = \boldsymbol{\theta}^T \boldsymbol{\delta},$$

Where $\boldsymbol{\theta}=[\theta_1,\theta_2,\ldots,\theta_N]^{\mathrm{T}}$ and $\boldsymbol{\delta}=[\delta_1,\delta_2,\ldots,\delta_N]^{\mathrm{T}}$ are regression vectors where each element is in the form of,

$$\delta_r = \frac{w_r}{\sum_{r=1}^{N} w_r} \quad (10)$$

In the present work, we have chosen Gaussian membership functions for the fuzzification process of s. The mathematical description of a general Gaussian function is in the form of,

$$\mu(s,\sigma,c) = \exp\left\{-\left[(s-c)^2 / \left(2\sigma^2\right)\right]\right\} \quad (11)$$

where c represents the center of the membership function and σ determines its width. In the extreme case, when the width is zero, the logic reduces to crisp logic.

According to the universal approximation property of the fuzzy systems, [14], (7) can be approximated by a fuzzy system with a bounded approximation error ε with a bound of E such as,

$$u^*(t) = u_{\mathrm{fz}}^* + \varepsilon = \boldsymbol{\theta}^{*\mathrm{T}} \boldsymbol{\delta} + \varepsilon \quad (12)$$

Let $\breve{u}_{\mathrm{fz}} = \breve{\boldsymbol{\theta}}^{\mathbf{T}} \boldsymbol{\delta}$ be the approximation of $u^*(t)$ and $\breve{\boldsymbol{\theta}}$ is the approximation of $\boldsymbol{\theta}^*$. Moreover, in order to compensate the approximation error $\breve{u}_{\mathrm{fz}} - u^*$, we add a variable structure controller term to the control signal which results,

$$u(t) = \breve{u}_{\mathrm{fz}} + u_{\mathrm{vs}} \quad (13)$$

Then the closed loop system is obtained as,

$$\begin{aligned} &\ddot{z}_s(t) = A_p \dot{z}_s(t) + K_p z_s(t) + D_p T_d \\ &+ M_p\left[\breve{u}_{\mathrm{fz}} + u_{\mathrm{vs}}\right] \end{aligned} \quad (14)$$

After some algebraic manipulations one, can achieve,

$$\ddot{e} + k_1 \dot{e} + k_2 e = M_p\left[\breve{u}_{\mathrm{fz}} + u_{\mathrm{vs}} - u^*\right] = \dot{s} \quad (15)$$

Denoting $\breve{u}_{\mathrm{fz}} - u^* = \breve{u}_{\mathrm{fz}} - u_{\mathrm{fz}}^* - \varepsilon$ as $\widehat{u}_{\mathrm{fz}}$ and $\breve{\boldsymbol{\theta}} - \boldsymbol{\theta}^*$ as $\widehat{\boldsymbol{\theta}}$ then,

$$\widehat{u}_{\mathrm{fz}} = \widehat{\boldsymbol{\theta}}^{\mathrm{T}} \boldsymbol{\delta} - \varepsilon \quad (16)$$

is obtained. The second term in control signal is the variable structure controller which is defined as

$$u_{\mathrm{vs}} = -\rho_1 \mathrm{sat}(s/\Phi) \quad (17)$$

where ρ_1 is the variable switching gain and Φ is the thickness of the boundary layer and chosen as 0.0008 for the present application and sat is the saturation function of the form,

$$\mathrm{sat}(s/\Phi) = \begin{cases} -1 & \text{if} \quad s/\Phi \le -1 \\ s/\Phi & \text{if} \quad -1 < s/\Phi \le 1 \\ 1 & \text{if} \quad s/\Phi > 1 \end{cases} \quad (18)$$

Let ρ_1^* is the equivalent gain of the sliding controller and $\breve{\rho}_1$ is the estimated gain. Then, the error of switching gain estimation is defined as,

$$\widehat{\rho}_1 = \breve{\rho}_1 - \rho_1^* \quad (19)$$

In order to achieve minimum approximation error and to guarantee the existence of sliding mode, we have chosen a Lyapunov function candidate as,

$$V(s,\widehat{\boldsymbol{\theta}},\widehat{\rho}_1) = \frac{1}{2}s^2 + \frac{M_p}{2\rho_1}\widehat{\boldsymbol{\theta}}^{\mathrm{T}}\widehat{\boldsymbol{\theta}} + \frac{M_p}{2\rho_2}\widehat{\rho}_1^2 \quad (20)$$

where ρ_2 is a positive constant. The time derivative of V along the closed-loop trajectory is,

$$\begin{aligned} &\dot{V}(s,\widehat{\boldsymbol{\theta}},\widehat{\rho}_1) \\ &= s\dot{s} + \frac{M_p}{\breve{\rho}_1}\widehat{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\widehat{\boldsymbol{\theta}}} + \frac{M_p}{\rho_2}\widehat{\rho}_1\dot{\widehat{\rho}}_1 \\ &= sM_p\left[\widehat{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\delta} + u_{\mathrm{vs}} - \varepsilon\right] + \frac{M_p}{\breve{\rho}_1}\widehat{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\widehat{\boldsymbol{\theta}}} + \frac{M_p}{\rho_2}\widehat{\rho}_1\dot{\widehat{\rho}}_1 \end{aligned}$$

$$= M_p \hat{\boldsymbol{\theta}}^{\mathrm{T}} \left[s\boldsymbol{\delta} + \frac{1}{\breve{\rho}_1} \dot{\hat{\boldsymbol{\theta}}} \right] + sM_p \left[-\breve{\rho}_1 \mathrm{sat}(s/\Phi) - \varepsilon \right] + \frac{M_p}{\rho_2} \left[\breve{\rho}_1 - \rho_1 \right] \dot{\breve{\rho}}_1 \tag{21}$$

In order to achieve, $\dot{V} < 0$ the adaptation law for $\dot{\breve{\rho}}_1$ is chosen as,

$$\dot{\breve{\rho}}_1 = \rho_2 s \mathrm{sat}(s/\Phi) \tag{22}$$

then (21) can be rewritten as

$$\begin{aligned} &\dot{V}(s, \hat{\boldsymbol{\theta}}, \breve{\rho}_1) \\ &= sM_p \left[-\breve{\rho}_1 \mathrm{sat}(s/\Phi) - \varepsilon \right] + M_p \left[\breve{\rho}_1 - \rho_1 \right] s \mathrm{sat}(s/\Phi) \\ &= -\varepsilon s M_p - \rho_1 s \mathrm{sat}(s/\Phi) M_p \leq |\varepsilon||s| M_p - \rho_1 |s| M_p \\ &= -M_p |s| \left(\rho_1 - |\varepsilon| \right) \leq 0. \end{aligned} \tag{23}$$

Defining $M_p |s| \left[\rho_1 - |\varepsilon| \right]$ as, $\psi(t)$ then

$$\psi(t) \leq -\dot{V}(s, \hat{\boldsymbol{\theta}}) \tag{24}$$

is valid. Since $\dot{V}\left(s(0), \hat{\boldsymbol{\theta}}\right)$ is bounded and $\dot{V}\left(s(t), \hat{\boldsymbol{\theta}}\right)$ is bounded and non-increasing, then, one can easily conclude that

$$\int_0^t \psi(\tau) d\tau < \infty \tag{25}$$

Since (25) is valid as t→∞ and ψ(t) is uniformly continuous, based on Barbalat's Lemma, [15], ψ(t)→∞ as t→∞. That is *s(t)*→∞ as t→∞ Thus, the stability of the proposed controller and adaptation laws are achieved in the sense of Lyapunov.

4. Simulation Results

To evaluate the proposed controller presented above, a simulation environment of AASS with the parameters in Table 1 is created for the quarter car model which has component-wise nonlinearities within. Damper and spring element nonlinearities as in original ones is used in the system dynamics equations. Vehicle speed of 36 km/h is chosen to emulate normal driving conditions and two types of road profiles are prepared for controller performance evaluation: standard bump-type surface profile with 10cm length x 10cm height and a random road profile generated to simulate stabilized road with 1cmx1cm pebbles. Open loop, PID and Proportional-Derivative (PD) type self-tuning fuzzy controllers are employed along with the proposed controller. Feedback of these evaluatory controllers are obtained through suspension deflection while proposed controller feedback signal is directly z_s, car static equilibrium position. The reason behind this difference is that evaluatory controllers do not provide reasonable system control with provided z_s feedback. The block diagram of the PD-type self-tuning fuzzy controller includes a main controller which processes the error and the error derivative values and the self-tuning mechanism that adjusts the gain matrix. The rule-bases for the main FLC and self-tuning mechanism are chosen, described in [15]. On the other hand, PID controller is tuned with well-known Ziegler-Nichols tuning method. Although the initial rule table of the proposed controller is not important because of the self organizing structure of the proposed controller, initial value of the adjustable vector θ is chosen as $\boldsymbol{\theta} = [5000 \;\; 3000 \;\; 1000 \;\; 0 \;\; -1000 \; -3000 \;\; 5000]^{\mathrm{T}}$, the values of k_1 and k_2 are chosen as 10.1 and 0.16, respectively.

First, bump-type road profile is applied to the system for four types of controllers and results are given in Figure 2, Figure 3 and Figure 4. Although the car body acceleration in vertical direction is the main target to reduce, car body displacement and suspension deflection are plotted for the ease of understanding. Car body displacement represents the system performance better than the other parameters, while suspension deflection illustrates the actuator response to road imperfections.

In Figure 2, proposed controller apparently produces the shortest response time of 0.85 sec. and the lowest peak value of 0.4 cm. Open loop response has continuing oscillations of 25 sec and also it has high peak value. PID controller decreases the peak value while decreasing the response time. On the other hand, STFPD still has high peak value and low response time comparing to the proposed controller. However, for chosen PD parameters, classical PID seems to perform better than PD type self-tuning fuzzy controller.

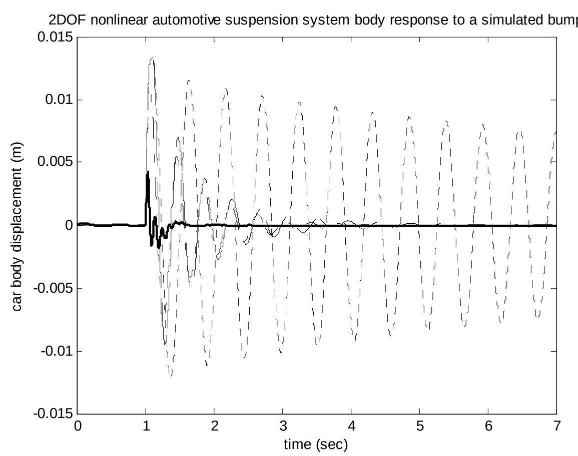

Figure 2. Car Body Displacement for a simulated bump (Proposed controller: solid bold; passive suspension: dotted; PID controller: dash-dot; STFPD controller: dash)

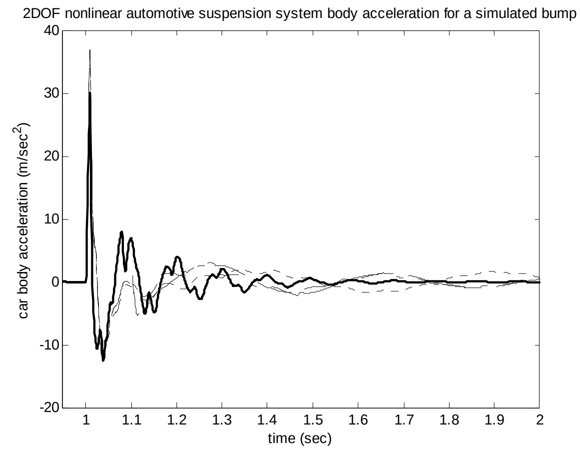

Figure 3. Car body acceleration for a simulated bump (Proposed controller: solid bold; passive suspension: dotted; PID controller: dash-dot; STFPD controller: dash)

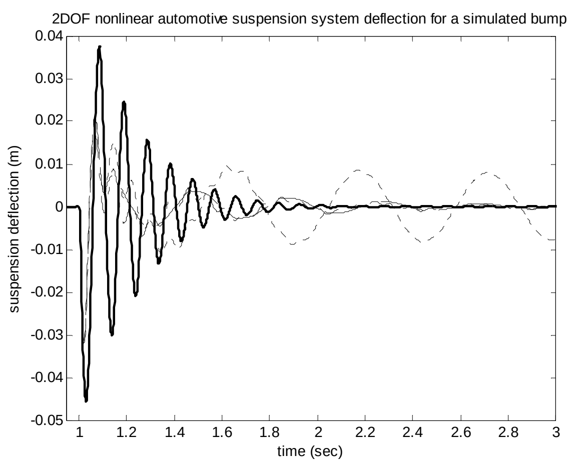

Figure 4. Suspension deflection for a simulated bump (Proposed controller: solid bold; passive suspension: dotted; PID controller: dash-dot; STFPD controller: dash)

The controller design purpose as indicated at the beginning of this paper is to decrease the body acceleration because of its effect on ride comfort. Comparing to other controllers listed, the proposed controller reduces peak acceleration value and response time drastically as shown in the figure. However, all three controllers have either high peak response or long response time for this type of road profile.

Because of the designed sliding mode controller and of chosen sliding trajectory, in case of the first road profile, the error and the error derivative follow the indicated trajectory till the set point on the sliding surface as shown in Figure 5. During this response, the actuator provides the highest possible control output when proposed controller is used, yet other controllers fail to sustain sufficient actuation.

Proposed controller and evaluatory ones has been tested using the random road profile. Figure 6 shows the response of all four controllers for this road profile and it is pretty obvious that the proposed sliding mode controller has overwhelming success over other controllers because of its quick response and provided ride comfort.

Additionally, acceleration response comparison for random road profile is shown in Figure 7 and the proposed controller has a unique response over other controllers. Summary of all responses to two kinds of road conditions can be seen through Table 2.

5. Conclusion

A novel single-input adaptive fuzzy sliding mode controller is proposed and successfully employed to control component-wise nonlinear AASS. The proposed scheme does not require any information from the controlled plant and any expert knowledge because of its learning capability. The strategy is robust since it has a single input FLC as a main controller. Thus, the rule base of the FLC drastically decreases when it is compared with the traditional FLCs. On the other hand, the efficiency of the controller is improved by combining a sliding mode compensator which also has an adaptive structure. The stability of the proposed scheme is achieved in the sense of Lyapunov. In order to demonstrate the effectiveness of the proposed method, the controller is applied to the suspension system in comparison with the passive suspension, PID controller, PD–type self-tuning fuzzy controller. Road profiles that are tested are a simulated random road surface and a bump. The experimental results show that the proposed scheme improves the ride comfort considerably when compared to the aforementioned controller structures. To demonstrate the efficacy of the controller and future study purposes, higher order geometric and component-wise nonlinear vehicle models will be approached for further research.

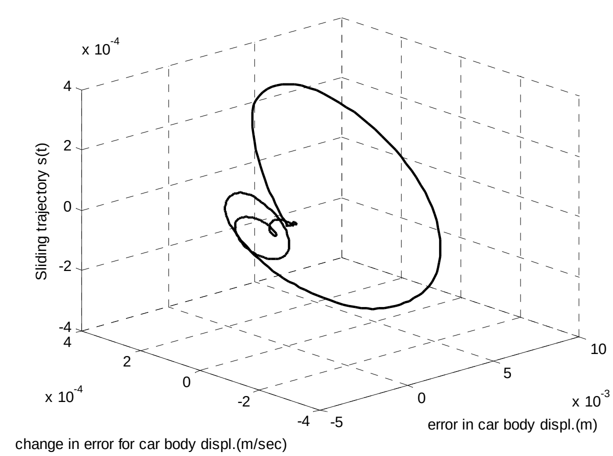

Figure 5. Control trajectories for proposed controller

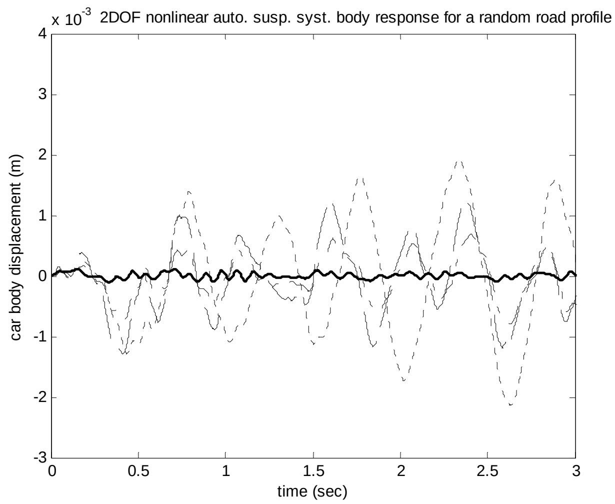

Figure 6. Body displacement for random road profile (Proposed controller: solid bold; passive suspension: dotted; PID controller: dot-dash; STFPD controller: dash)

Table 1. Two DOF car model system parameters

Parameters	Definition	Value
m_s	Car body mass	250.3 kg
m_u	Wheel mass	30.41 kg
k_s	Suspension spring constant	45000 N/m
k_t	Tire spring constant	150000 N/m
b_s	Suspension damping coefficient	1000 N/(m/sec)
v	Vehicle speed	72 km/h

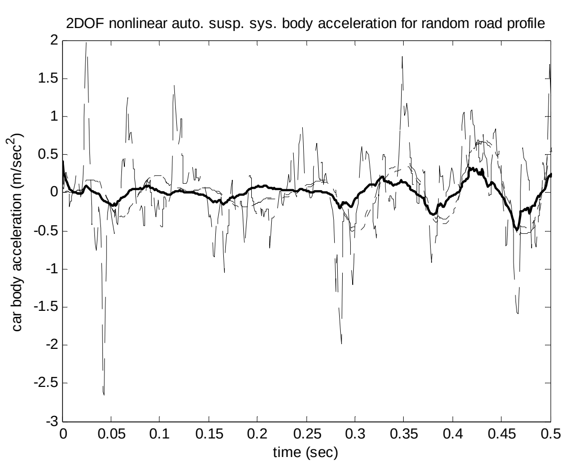

Figure 7. Car body acceleration for random road profile (a:Proposed controller, b: passive suspension, c:PID controller, d:STFPD controller)

Table 2. Comparison of the controller performances

Controller	Body acc. - random road profile (RMS)	Body acc-simulated bump (RMS)	First Peak - simulated bump(m)	Sett. Time - simulated bump (sec.)
Proposed	*0.1288*	*1.2538*	*0.004*	*0.85*
Passive (Open-loop)	0.4994	1.6369	0.0133	25
STFPD	0.3209	1.4876	0.0135	6
PID	0.2936	1.4097	0.0112	4.15

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