

## **A comparison study of robust control strategies for automotive active suspension systems**

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## A Comparison Study of Robust Control Strategies for Automotive Active Suspension Systems ( $H_\infty$ , LQR and Fuzzy Logic Control)

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### Abstract

*In practice, the sprung mass of a vehicle directly varies with the number of passengers and the baggage load. This variation negatively affects the control performance. In this respect, the control mechanism of the automotive active suspension system(AASS) that is used should be robust in nature. In the present work, the robustness properties of Linear Quadratic Regulator (LQR) and  $H_\infty$  controllers are compared with the fuzzy logic controller (FLC) which is widely used in intelligent robust systems. First of all LQR,  $H_\infty$  and FLCs are designed individually then simulated on a quarter car model with nominal parameters in order to investigate the nominal performances in time domain. Finally the robustness properties of the same controllers are investigated on the suspension model where the mass of the car is augmented 40% of its nominal value.*

### 1. Introduction

Since the disturbances and coercive forces originated from road causes noise and ride comfort problems on a vehicle, it is very important to investigate and control the vehicle vibration. A vehicle can be modeled as a complex multi-massed dynamic system where the complexity completely depends on the objective of the model used. Until now, three types of new generation suspensions have proposed in order to compensate for the vehicle vibrations. These are passive, semi active and active suspension systems. In this work, active suspension systems are considered.

During the evaluation process of vehicle vibrations, four important performance criteria are widely used. These are: ride comfort, road handling capability, suspension deflection and amount of energy expended. In order to increase the ride comfort, the movement and acceleration of body should be reduced. On the other hand, in order to increase the road handling capability, dynamic tire pressure of the vehicle should be maximized.

However, the suspension deflections are limited by constructive reasons and maximizing tire pressure deteriorates the ride comfort. In this respect, these three aforementioned criteria highly depend on each other. When compared with the other criteria mentioned above, the amount of energy used in the control mechanism is much more independent. In this work, our goal is the minimization of the four criteria discussed above simultaneously where the controller is not affected by external disturbances, thus a multi-objective controller is needed.

In vehicles, the sprung mass of a vehicle directly varies with the number of passengers and the weight. This situation negatively affects the control performance. In this respect, the controller that is used in active suspension systems should be robust in nature. First of all, LQR,  $H_\infty$  and fuzzy controllers are designed individually then simulated on a quarter car model with nominal parameters in order to investigate the performances in time domain. Finally the robustness properties of the same controllers are investigated on the suspension model where the mass of the car is augmented 40% of its nominal value.

### 2. Quarter Car model

The block diagram of the quarter car model used in the present work is shown in Figure 1. Here  $k_1=20000\text{N/m}$  and  $c_1=1000\text{Ns/m}$  denotes the spring constant and damping constant of the suspension, respectively. Whereas,  $k_2=200000\text{N/m}$  and  $c_2=0.015\text{Ns/m}$  stand for the spring constant and damping constant of the tire, respectively. On the other hand  $m_1=1000\text{kg}$  and  $m_2=150\text{kg}$  stand for the mass of car body and the wheel, respectively. Finally  $u$  is the control force applied to the system measured in Newton by the controller,  $x_1$  is the body displacement,  $x_2$  is the wheel displacement measured in meter.  $x_3$  is the velocity of the body and finally  $x_4$  stands for the velocity of the wheel. The system model is chosen as in [1].

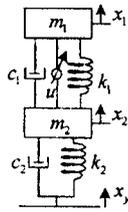


Figure 1. Quarter car model

### 3. Lqr design

The system model used throughout the present paper is continuous-time multi input multi output linear time invariant one. The plant is characterized by  $P$  and we assume a quadratic performance index in the form of

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (1)$$

where  $Q$  and  $R$  are symmetric positive definite matrices. Then it is well-known that the optimal feedback control law is  $u = -Kx$  where  $K = R^{-1} B^T X$  and  $X$  is positive definite solution of the Riccati equation  $PA + A^T P - PBR^{-1} B^T P + Q = 0$ . The plant  $P$  is given by the state equations

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned} \quad (2)$$

where  $w$ ,  $u$ ,  $z$  and  $y$  are the vectors of exogenous inputs, control inputs, regulated outputs and measured outputs, respectively. Thus the overall generalized feedback controlled system will be as shown in Figure 2.

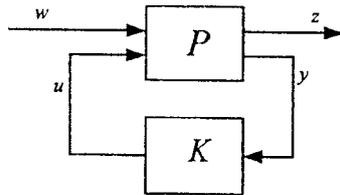


Figure 2. Generalized feedback structure

In the design of the LQR controller, weighting matrices  $Q$  and  $R$  are selected as  $Q = \text{diag}[100, 10000, 1000, 1000]$  and  $R = 0.000009$ . Thus the designed gain vector of the LQR controller is  $K = 10000 \cdot [-4.3355 \quad 8.7290 \quad -0.2067 \quad 1.4832]$ .

### 5. Flc design

The FLC that is used in the current paper is in PD (Proportional-Derivative) structure. As an implication method  $\min$  operator is used. On the other hand all  $t$  - norm operations are performed with  $\min$  operator and all  $s$  - norm operations are performed with  $\max$  operator. Also  $\min$  operator is

used in implication procedure. Besides, defuzzification method is chosen to be as center of averages. The input membership functions are shown in Figure 3, Figure 4, respectively. The output membership functions are chosen as triangular-form that are uniformly distributed between  $[-1, 1]$ . Scaling factors for inputs and outputs are chosen as follows:  $G_e = 50$ ,  $G_{\Delta e} = 10$ ,  $G_u = 1600$ . Finally, the rule base is shown in Figure 5. For the controller mechanism the feedback is taken from  $x_1$ .

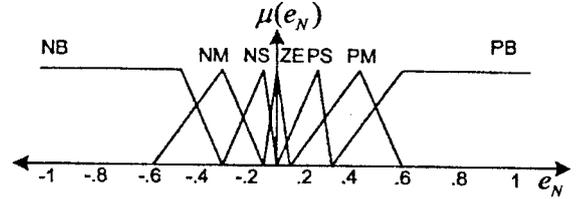


Figure 3. Fuzzification functions for  $e$

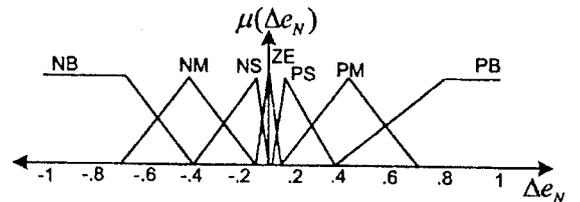


Figure 4. Fuzzification functions for  $\Delta e$

Table 1. Rule base for the FLC

$e$	NB	NM	NS	ZE	PS	PM	PB
NB	PB	PB	PM	PM	PS	ZE	ZE
NM	PB	PM	PM	PS	ZE	ZE	NS
NS	PM	PM	PS	ZE	ZE	NS	NS
ZE	PM	PS	ZE	ZE	NS	NS	NM
PS	PS	ZE	ZE	NS	NS	NM	NM
PM	ZE	ZE	NS	NS	NM	NM	NB
PB	ZE	NS	NS	NM	NM	NB	NB

### 6. $H_\infty$ controller design

For the generalized plant,  $w = x_y$ ,  $z = [x_1, x_1 - x_2, x_2, u]$  and the block diagram of the  $H_\infty$  controller is shown in Figure 5. Here  $W_{idi}$ 's  $i=0, \dots, 3$  represent the rational and stable scaling transfer functions for the inputs, and they are chosen as following:  $W_{id0} = 0.8(s+20)^{-1}$ ,  $W_{id1} = 10^{-5}$ ,  $W_{id2} = 10^{-4}$ ,  $W_{id3} = 10^{-9}$ . On the other hand,  $W_{tzi}$  represents the rational and stable scaling transfer functions for the  $z$  performance outputs. Here are the  $W_{tzi}$ 's:

$$W_{t1} = \frac{0.007958 \cdot s + 0.1}{0.0003166 \cdot s^2 + 0.03144 \cdot s + 1}, W_{t2} = 10, W_{t3} = 0.1,$$

$W_{t4} = 10^{-5}$ .  $d(i)$ 's,  $i=1,2,3$  are all chosen as white noise processes with power=0.1. The inputs for the  $H_\infty$  controller are  $x_1, x_1 - x_2$ , respectively. Finally the aim of the controller is the minimization of  $\|T_{zw}\|_\infty$ .

For the synthesis of the corresponding controller, MATLAB<sup>®</sup> is used.

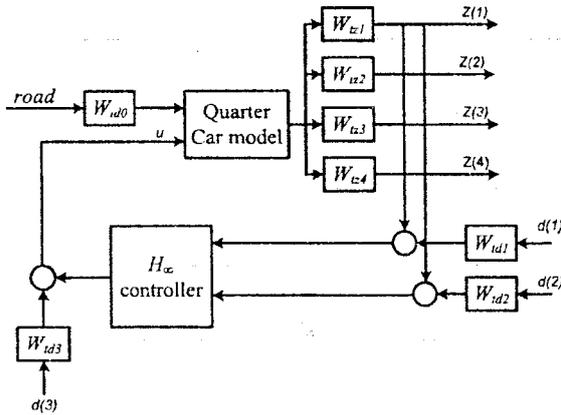


Figure 5. Generalized block diagram for the synthesis of  $H_\infty$  controller

## 7. Simulation results

All the experiments are carried out with a road function which is shown in Figure 6. The comparison between the controllers and the passive suspension mechanism are evaluated in terms of body acceleration, car body displacement, suspension deflection and applied force. Besides, in order to test the robustness of the previously mentioned controllers, the same experiments are repeated by increasing the mass of the car 40% of its nominal value. At the same time, the damping coefficient of the suspension is decreased 40% of its nominal value in order to get a realistic damping coefficient. It is well known that the damping coefficient of suspension systems decreases with the increasing operation time. First, the nominal performances are compared. Figure 7 shows the vertical displacement responses of the previously mentioned controllers. It is obvious that  $H_\infty$  controller shows the best performance.

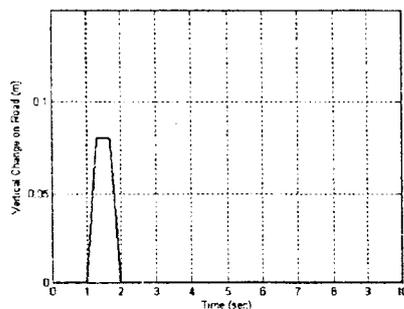


Figure 6. Road function applied to the system

Figure 8 shows the body acceleration of the car. This time best performance is achieved by the FLC. Finally suspension deflection comparison test is performed. Again  $H_\infty$  controller demonstrated the best performance. Figure 9 shows the suspension

deflections of the system in subject to the above mentioned controllers.

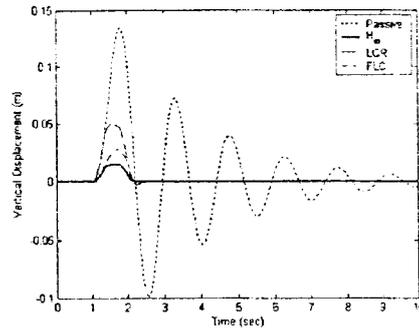


Figure 7. Vertical displacement responses for nominal system parameters

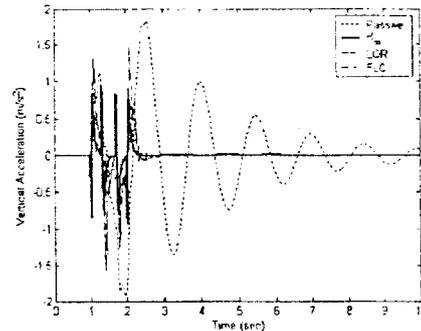


Figure 8. Car body accelerations with the nominal system parameters

As a second test robustness properties of the controllers are exposed. The acceleration, displacement of the car body, and suspension deflections of the controllers in subject to parameter variations are shown in Figure 10, Figure 11 and Figure 12, respectively.

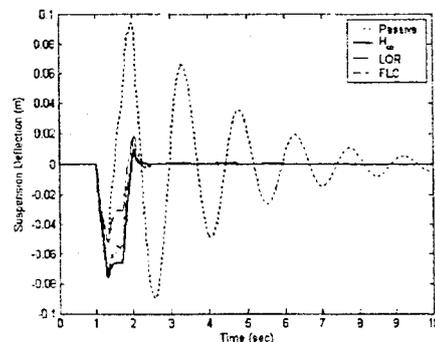


Figure 9. Suspension deflections for the controllers

Again for the acceleration performance, FLC showed the best performance. Whereas,  $H_\infty$  controller has shown the best response for the body displacement and suspension deflection measures.

In order to summarize the responses and to make an efficient comparison, the controller performances were collected in Table 3 and Table 4.

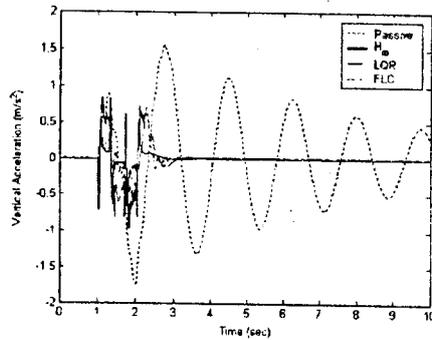


Figure 10. Vertical accelerations for the controllers

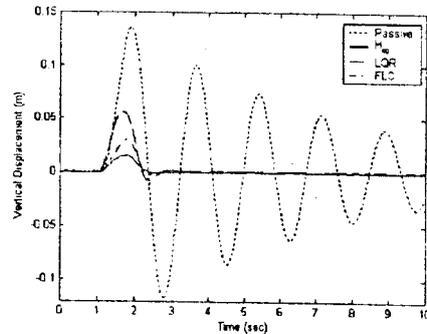


Figure 11. Vertical displacements for the controllers

Here, Table 3 presents the performances under nominal parameters whereas the tests in Table 4 were performed with the varied parameters.

Table 2. Performance results for nominal conditions

CASE 1	$H_\infty$	LQR	FLC	Passive
$\max(x_1-x_2)$	0.0855	0.0722	0.0914	0.1833
$\ x_1 - x_2\ $	0.2541	0.2525	0.2919	0.9450
$\max(u)$	2220.7	1878.7	1538.5	0
$\ u\ $	11496	6005	6425.8	0
$\max(x_i)$	0.0148	0.0501	0.0278	0.1346
$\ x_i\ $	0.1084	0.3663	0.1181	1.2382
$\max(a)$	0.9380	1.0121	0.7334	1.9337
$\ a\ $	4.3785	6.4068	2.5117	19.380

Table 3. Performance results for varied conditions

CASE 2	$H_\infty$	LQR	FLC	Passive
$\max(x_1-x_2)$	0.0923	0.0872	0.1001	0.2268
$\ x_1 - x_2\ $	0.2545	0.3451	0.3143	1.4326
$\max(u)$	2244.8	1523.5	1450.3	0
$\ u\ $	11474	10071	7015.4	0
$\max(x_i)$	0.0157	0.0558	0.0295	0.1365
$\ x_i\ $	0.1091	0.3961	0.1348	1.6760
$\max(a)$	0.8111	0.9596	0.6596	1.7354
$\ a\ $	3.2343	6.0000	2.3936	20.611

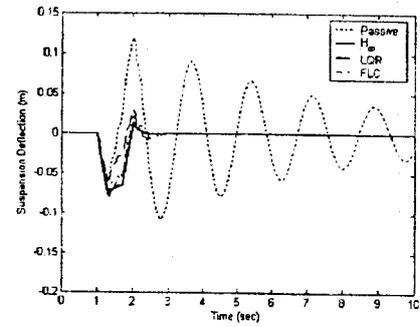


Figure 12. Suspension deflections for the controllers

In order to bring the selection procedure to a conclusion, a new performance index is proposed. This index is in the form of

$$P(\ddot{x}_{1N}, \dot{x}_{1N}, x_{1N} - x_{2N}, u_N) = a_1 \ddot{x}_{1N} + a_2 \dot{x}_{1N} + a_3 (x_{1N} - x_{2N}) + a_4 u_N \quad (3)$$

Here,  $a_i, i=1, \dots, 4$  represents the weights. Besides, the variables are in normalized form. In this work, the weights were chosen as:  $a_1=0.3, a_2=0.3, a_3=0.3$  and  $a_4=0.1$ . The performance results that are obtained are summarized in Table 4. Now one can easily to conclude that the best controller for AASS is  $H_\infty$  controller.

One can get a different performance index by changing the weights in (3) in order to fulfill his needs. For example, for a coach, ride comfort consequently the minimization of body acceleration would be much more important than for an off-road vehicle.

Table 4. Performance results for the new criteria

	$H_\infty$	FLC	LQR
Case 1	0.7400	0.7565	0.9370
Case 2	0.6991	0.7339	0.9613

## 8. Conclusion

In this work, robust control strategies that are widely used in control community are compared for the control of AASS. Simulation results demonstrate that  $H_\infty$  optimal controller shows the best performance in both nominal and varying conditions. Besides, it is very surprising that a simple well-tuned FLC demonstrates better performance than an LQR.

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