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RELIABILITY ANALYSIS OF A RODDING ANODE PLANT IN ALUMINUM INDUSTRY WITH MULTIPLE UNITS FAILURE AND SINGLE REPAIRMAN

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Abstract – The paper presents reliability analysis of a rodding anode plant in aluminum industry with multiple unit failure and single repairman. Manufacturing process of raw aluminum blocks in this plant passes through eight stations viz., butt shot blast station 1 with three components, butt & thimble removal press station 2 with standby arrangement, and here each machine consists of two components, combined btp (butt & thimble press) station 3 with one component, stub straighten station 4 with one component, stub shot blast station 5 with two components, stub coating and drying station 6 with two components, casting station 7 with four components, and anode rod inspection station 8 with one component. Failure of any of the stations brings the system to a complete halt, except the butt & thimble removal press stations because of the parallel standby arrangement, and does not affect the system operation completely unless both the units fail. Six years of maintenance data on component failures, repairs and associated costs are used in this analysis. Measures of system effectiveness is gauged through reliability indices such as mean time to plant failure (MTPF), availability of the plant, busy period of repairman and expected number of repairs. Effect of repair rate, failure rate and repair cost on system performance w.r.t. revenue is shown graphically. Theory of Semi-Markov and regenerative stochastic processes is used in the analysis.

Keywords - Reliability, semi-Markov, regenerative point, failures, repairs, rodding anode plant

1 Notations and symbols for the state of the system

O_i	State i is operative
D_i	State i is Down
λ_i	Estimated of failure rate of i^{th} unit
α_i	Estimated of repair rate of i^{th} unit
$F_{i\,r}$	j^{th} unit of i^{th} station is under repair
$F_{i\,W\,r}$	j^{th} unit of i^{th} station is waiting for repair
$F_{i\,R}$	j^{th} unit of i^{th} station is continuing for repair from the previous state
$p_i, p^{(k)}{}_i$	Probability of transition from a regenerative state i to a regenerative state j without visiting any other state in $(0,t]$, probability of transition from a regenerative state i to a regenerative state j via state k state *(0,t]*
$*$/*LT*	Symbol of Laplace transform
$**$/*LST*	Symbol of Laplace-Steiltje's transform
$m_i, m^{(k)}{}_i$	The unconditional mean time taken to transit to any regenerative state from the epoch of entery into regenerative state j without visiting any failed states, visiting failed state k once
μ_i	Sojourn time in the regenerative state i
©	Laplace convolution
Ⓢ	Steiltje'sconvolution
$\phi_0(t)$	Cumulative distribution function $c.d.f$ of the first passage time from a regenerative state i to a failed state
$A_i(t)$	The probability of the unit entering into upstate at instant t, giving that the unit entered in regenerative state i at $t\ = 0$
$B_i(t)$	Probability that the repairman is busy in inspection of instant t, given that the system entered regenerative state i at $t = 0$
$V_i(t)$	Expected number of visits of the repairman, given that the system entered regenerative state i at $t = 0$
$M_i(t)$	The probability that the system initially up in regenerative state i, is up at a time t without going to any regenerative state
$W_i(t)$	Probability that that the repairman is busy in regenerative state i at time t without passing any other regenerative state
$p.d.f, c.d.f$	Probability density function, Cumulative distribution function

2 Introduction

Aluminum manufacturing industries play a major role in economic growth of the country. Due to increasing demand of the Aluminum and its byproducts, industrial systems catering to these requirements need to be maintained efficiently. Efficient maintenance attributes in achieving optimum system reliability and further helps in avoiding big loses. Researchers have spent a great deal of efforts in the past to address the issues arising in complex system maintenance, by analyzing the hypothetical and real operating situations of the industrial systems, from reliability perspective. Authors including (Attahiru et al. 1998; Ram et al. 2013; Niwas et al. 2014) have analyzed repairable system failure with three units, standby system with waiting repair, and inspection for feasibility or repair beyond warranty. Continuous casting plant of steel manufacturing industry was extensively analyzed for different operating situations (Mathew et al. 2009, 2010, 2011). Many case studies on industrial systems with different failure and repair situations have been reported in the literature (Rizwan et al. 2005, 2006, 2007, 2010, 2013, 2014, 2015). On similar lines, analyses pertaining to desalination plant have been reported by Padmavathi et al. (2013, 2014, 2015). Standby system with server failure, redundant system with standby failure and assuming arbitrary distribution for repair and replacement times were considered by Bhardwaj and Singh (2014), Bhardwaj et al. (2017). Later, Taj et al. (2017) used similar modeling methodology for system analysis for a cable plant with six maintenance categories. Thus, the methodology for industrial system analysis has been widely presented in the literature and proved to be a good tool for industrial system analysis, and therefore the novelty of this work lies in its case study for a different system with different operating conditions. Recently, the methodology was further extended by Yaqoob Al Rahbi et al. (2017) for analyzing a system of rodding anode plant in Aluminum industry. Aluminum manufacturing process passes through eight stations viz., butt shot blast station 1, butt & thimble removal press station 2 with standby arrangement, combined btp (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. Analysis in this case is carried out for a system containing eight stations considering each station as a single unit. This seems to be a basic and simple operating situation, and opens up a further scope of complex situation which is quite realistic from possible system failure risks associated to all units operating at different stations. There are three units at station 1, two units at station 2; 3rd, 4th and 8th stations have one unit each; 5th station has two units, 6th station has again two units and 7th station has four units. Thus, the present analysis portrays a multiple unit failure situations of the plant, and obtains reliability indices reflecting the overall system performance. Six years of maintenance data are used in this analysis. Failure, repair rates of the units, and various associated costs are estimated from the data. The plant operates round the clock, and the failure at any of the stations impacts the plant to a shutdown situation, except station 2 which has a standby arrangement and do not affect the system operation completely unless both the units at this station fail.

The system is analyzed by using semi-Markov processes (Ibe 2008) and regenerative stochastic processes (Smith 1955, 1958), and the following expressions of the reliability indices are obtained:

- Mean Time to Plant Failure (MTPF)
- Steady State Availability (A_0)
- Busy period of the repairman (B_0)
- Expected number of visits by the repairman (V_0)
- System Profit (P)

3 Model descriptions and assumptions

The following are the descriptions and assumption of the model:

1. Initially the plant is operational at state 0 with all stations and all units working.
2. All necessary maintenances are off-line which means plant need to be in shut down state for all repairs or replacements.
3. Maintenances are all random and need to be addressed on requirement by a single repairman.
4. All failure times are assumed to have exponential distribution whereas other times are general.
5. After each repair, the plant works as good as new and returns to new state.
6. Repairman comes as soon as a unit fails, and all other failures need to wait until previous failures have been resolved.

The model has the following mutually exclusive states of the system, using renewal theory (Cox 1962):

Regenerative states:

$S_0 = (O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8);$

$S_1 = (F_{1r,1} D_2, D_3, D_4, D_5, D_6, D_7, D_8);$

$S_2 = (F_{1r,2} D_2, D_3, D_4, D_5, D_6, D_7, D_8);$

$S_3 = (F_{1r,3} D_2, D_3, D_4, D_5, D_6, D_7, D_8);$

$S_4 = (O_1, O_{2F2r',1} O_3, O_4, O_5, O_6, O_7, O_8);$

$S_5 = (O_1, O_{2F2r',2} D_3, D_4, D_5, D_6, D_7, D_8);$

$S_6 = (D_1, D_2, F_{3r,1} D_4, D_5, D_6, D_7, D_8);$

$S_7 = (D_1, D_2, D_3, F_{4r,1} D_5, D_6, D_7, D_8)$

$S_8 = (D_1, D_2, D_3, D_4, F_{5r,1} D_6, D_7, D_8);$

$S_9 = (D_1, D_2, D_3, D_4, F_{5r,2} D_6, D_7, D_8);$

$S_{10} = (D_1, D_2, D_3, D_4, D_5, F_{6r,1} D_7, D_8);$

$S_{11} = (D_1, D_2, D_3, D_4, D_5, F_{6r,2} D_7, D_8);$

$S_{12} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7r,1} D_8);$

$S_{13} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7r,2} D_8);$

$S_{14} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7r,3} D_8);$

$S_{15} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7r,4} D_8);$

$S_{16} = (D_1, D_2, D_3, D_4, D_5, D_6, D_7, F_{8r});$

Non-regenerative states:

$S_{17} = (F_{1Wr1}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{18} = (F_{1Wr2}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{19} = (F_{1Wr3}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{20} = (D_1, D_{2F2Wr1}, O_3, O_4, O_5, O_6, O_7, O_8)$;
$S_{21} = (D_1, D_{2F2Wr2}, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{22} = (D_1, D_2, F_{3Wr1}, D_4, D_5, D_6, D_7, D_8)$;
$S_{23} = (D_1, D_2, D_3, F_{4Wr1}, D_5, D_6, D_7, D_8)$;
$S_{24} = (D_1, D_2, D_3, D_4, F_{5Wr1}, D_6, D_7, D_8)$;
$S_{25} = (D_1, D_2, D_3, D_4, F_{5Wr2}, D_6, D_7, D_8)$;
$S_{26} = (D_1, D_2, D_3, D_4, D_5, F_{6Wr1}, D_7, D_8)$;
$S_{27} = (D_1, D_2, D_3, D_4, D_5, F_{6Wr2}, D_7, D_8)$;
$S_{28} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7Wr1}, D_8)$;
$S_{29} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7Wr2}, D_8)$;
$S_{30} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7Wr3}, D_8)$;
$S_{31} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7Wr4}, D_8)$;
$S_{32} = (D_1, D_2, D_3, D_4, D_5, D_6, D_7, F_{8W})$;
$S_{33} = (F_{1R1}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{34} = (F_{1R2}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{35} = (F_{1R3}, D_2, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{36} = (D_1, D_{2F2R1}, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{37} = (D_1, D_{2F2R2}, D_3, D_4, D_5, D_6, D_7, D_8)$;
$S_{38} = (D_1, D_2, F_{3R1}, D_4, D_5, D_6, D_7, D_8)$;
$S_{39} = (D_1, D_2, D_3, F_{4R1}, D_5, D_6, D_7, D_8)$;
$S_{40} = (D_1, D_2, D_3, D_4, F_{5R1}, D_6, D_7, D_8)$;
$S_{41} = (D_1, D_2, D_3, D_4, F_{5R2}, D_6, D_7, D_8)$;
$S_{42} = (D_1, D_2, D_3, D_4, D_5, F_{6R1}, D_7, D_8)$;
$S_{43} = (D_1, D_2, D_3, D_4, D_5, F_{6R2}, D_7, D_8)$;
$S_{44} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7R1}, D_8)$;
$S_{45} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7R2}, D_8)$;
$S_{46} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7R3}, D_8)$;
$S_{47} = (D_1, D_2, D_3, D_4, D_5, D_6, F_{7R4}, D_8)$;
$S_{48} = (D_1, D_2, D_3, D_4, D_5, D_6, D_7, F_{8R})$;

The data summary of the system reflects the following estimates:

Table I Estimated values for the plant

Units	Failure rate	Repair rate	Average Cost in Omani Riyal (OMR)
1.	$\lambda_1 = 0.01614$	$\alpha_1 = 0.21269$	240.8
2.	$\lambda_2 = 0.00147$	$\alpha_2 = 0.21812$	1113.0
3.	$\lambda_3 = 0.00416$	$\alpha_3 = 0.22885$	320.4
4.	$\lambda_4 = 0.01971$	$\alpha_4 = 0.17785$	546.0
5.	$\lambda_5 = 0.01706$	$\alpha_5 = 0.17962$	600.8
6.	$\lambda_6 = 0.00053$	$\alpha_6 = 0.23088$	57.8
7.	$\lambda_7 = 0.00192$	$\alpha_7 = 0.51622$	2727.7
8.	$\lambda_8 = 0.00289$	$\alpha_8 = 0.18881$	298.1
9.	$\lambda_9 = 0.00092$	$\alpha_9 = 0.16331$	115.9
10.	$\lambda_{10} = 0.00317$	$\alpha_{10} = 0.23818$	397.5
11.	$\lambda_{11} = 0.00124$	$\alpha_{11} = 0.21487$	538.4
12.	$\lambda_{12} = 0.01931$	$\alpha_{12} = 0.18025$	252.9
13.	$\lambda_{13} = 0.00012$	$\alpha_{13} = 0.20660$	216.6
14.	$\lambda_{14} = 0.00217$	$\alpha_{14} = 0.20254$	187.4
15.	$\lambda_{15} = 0.00009$	$\alpha_{15} = 0.17115$	149.9
16.	$\lambda_{16} = 0.00058$	$\alpha_{16} = 0.17198$	373.3

4 Transition probabilities and mean sojourn times

Considering various transition states of the system, following non-zero elements $p_{ij} \geq 0$; from state i to state j are obtained, where $p_{ij} = \lim_{s \to 0} \int_0^\infty q_{ij}(t)\,dt$

$p_{01} = \frac{\lambda_1}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{02} = \frac{\lambda_2}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{03} = \frac{\lambda_3}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{04} = \frac{\lambda_4}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{05} = \frac{\lambda_5}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{06} = \frac{\lambda_6}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{07} = \frac{\lambda_7}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{08} = \frac{\lambda_8}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{09} = \frac{\lambda_9}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{0,10} = \frac{\lambda_{10}}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{0,11} = \frac{\lambda_{11}}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{0,12} = \frac{\lambda_{12}}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{0,13} = \frac{\lambda_{13}}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{0,14} = \frac{\lambda_{14}}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{0,15} = \frac{\lambda_{15}}{\lambda_1 + \cdots + \lambda_{16}}$ $\quad p_{0,16} = \frac{\lambda_{16}}{\lambda_1 + \cdots + \lambda_{16}}$

$p_{10} = g_1^*(0)$ $\quad p_{20} = g_2^*(0)$

$p_{30} = g_3^*(0)$ $\quad p_{40} = g_4^*(0)$

$p_{50} = g_5^*(0)$ $\quad p_{60} = g_6^*(0)$

$p_{70} = g_7^*(0)$ $\quad p_{80} = g_8^*(0)$

$p_{90} = g_9^*(0)$ $\quad p_{10,0} = g_{10}^*(0)$

$p_{11,0} = g_{11}^*(0)$ $\quad p_{12,0} = g_{12}^*(0)$

$p_{13,0} = g_{13}^*(0)$ $\quad p_{14,0} = g_{14}^*(0)$

$p_{15,0} = g_{15}^*(0)$ $\quad p_{16,0} = g_{16}^*(0)$

$p_{4,17} = \frac{\lambda_1}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_1 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,18} = \frac{\lambda_2}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_2 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,19} = \frac{\lambda_3}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_3 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,20} = \frac{\lambda_4}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_4 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,21} = \frac{\lambda_5}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_5 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,22} = \frac{\lambda_6}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_6 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,23} = \frac{\lambda_7}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_7 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,24} = \frac{\lambda_8}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_8 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,25} = \frac{\lambda_9}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_9 g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,26} = \frac{\lambda_{10}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{10} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,27} = \frac{\lambda_{11}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{11} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,28} = \frac{\lambda_{12}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{12} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,29} = \frac{\lambda_{13}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{13} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,30} = \frac{\lambda_{14}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{14} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,31} = \frac{\lambda_{15}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{15} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{4,32} = \frac{\lambda_{16}}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_{16} g_4^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{5,33} = \frac{\lambda_1}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_1 g_5^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{5,34} = \frac{\lambda_2}{\lambda_1 + \cdots + \lambda_{16}} - \lambda_2 g_5^*(\lambda_1 + \cdots + \lambda_{16})$

$p_{5,35}=\frac{\lambda_3}{\lambda_1+\cdots+\lambda_{16}}-\lambda_3 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,36}=\frac{\lambda_4}{\lambda_1+\cdots+\lambda_{16}}-\lambda_4 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,37}=\frac{\lambda_5}{\lambda_1+\cdots+\lambda_{16}}-\lambda_5 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,38}=\frac{\lambda_6}{\lambda_1+\cdots+\lambda_{16}}-\lambda_6 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,39}=\frac{\lambda_7}{\lambda_1+\cdots+\lambda_{16}}-\lambda_7 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,40}=\frac{\lambda_8}{\lambda_1+\cdots+\lambda_{16}}-\lambda_8 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,41}=\frac{\lambda_9}{\lambda_1+\cdots+\lambda_{16}}-\lambda_9 g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,42}=\frac{\lambda_{10}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{10} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,43}=\frac{\lambda_{11}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{11} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{4,44}=\frac{\lambda_{12}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{12} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,45}=\frac{\lambda_{13}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{13} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,46}=\frac{\lambda_{14}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{14} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,47}=\frac{\lambda_{15}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{15} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{5,48}=\frac{\lambda_{16}}{\lambda_1+\cdots+\lambda_{16}}-\lambda_{16} g_5^*(\lambda_1+\cdots+\lambda_{16})$
$p_{41}^{(17)}=\frac{\lambda_1}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{42}^{(18)}=\frac{\lambda_2}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{43}^{(19)}=\frac{\lambda_3}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{44}^{(20)}=\frac{\lambda_4}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{44}^{(21)}=\frac{\lambda_5}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{46}^{(22)}=\frac{\lambda_6}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{47}^{(23)}=\frac{\lambda_7}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{48}^{(24)}=\frac{\lambda_8}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{49}^{(25)}=\frac{\lambda_9}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,10}^{(26)}=\frac{\lambda_{10}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,11}^{(27)}=\frac{\lambda_{11}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,12}^{(28)}=\frac{\lambda_{12}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,13}^{(29)}=\frac{\lambda_{13}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,14}^{(30)}=\frac{\lambda_{14}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,15}^{(31)}=\frac{\lambda_{15}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{4,16}^{(32)}=\frac{\lambda_{16}}{\lambda_1+\cdots+\lambda_{16}}[g_4^*(0)-g_4^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{51}^{(33)}=\frac{\lambda_1}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{52}^{(34)}=\frac{\lambda_2}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{53}^{(35)}=\frac{\lambda_3}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{55}^{(36)}=\frac{\lambda_4}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{55}^{(37)}=\frac{\lambda_5}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{56}^{(38)}=\frac{\lambda_6}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{57}^{(39)}=\frac{\lambda_7}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{58}^{(40)}=\frac{\lambda_8}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{59}^{(41)}=\frac{\lambda_9}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,10}^{(42)}=\frac{\lambda_{10}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,11}^{(43)}=\frac{\lambda_{11}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,12}^{(44)}=\frac{\lambda_{12}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,13}^{(45)}=\frac{\lambda_{13}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,14}^{(46)}=\frac{\lambda_{14}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,15}^{(47)}=\frac{\lambda_{15}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$
$p_{5,16}^{(48)}=\frac{\lambda_{16}}{\lambda_1+\cdots+\lambda_{16}}[g_5^*(0)-g_5^*(\lambda_1+\cdots+\lambda_{16})]$

By these transition probabilities, it can be verified that

$p_{01}+\cdots+p_{0,16}=1$
$p_{10}=p_{20}=p_{30}=p_{60}=p_{70}=\cdots=p_{160}=1$
$p_{40}+p_{4,17}+\cdots+p_{4,32}=1$
$p_{50}+p_{5,33}+\cdots+p_{5,48}=1$
$p_{40}+p_{41}^{(17)}+\cdots+p_{4,16}^{(32)}=1$
$p_{50}+p_{51}^{(33)}+\cdots+p_{5,16}^{(48)}=1$

The unconditional mean time taken by the system to transit for any state j when it has taken from epoch of entrance into regenerative state i is mathematically stated as:

$m_{ij}=\int_0^\infty t\,Q_{ij}(t)\,dt$, or $\lim_{s\to 0}-\frac{d}{ds}\left(q_{ij}^*(s)\right)$

$m_{01}=\frac{\lambda_1}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{02}=\frac{\lambda_2}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{03}=\frac{\lambda_3}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{04}=\frac{\lambda_4}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{05}=\frac{\lambda_4}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{06}=\frac{\lambda_6}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{07}=\frac{\lambda_7}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{08}=\frac{\lambda_8}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{09}=\frac{\lambda_9}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{0,10}=\frac{\lambda_{10}}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{0,11}=\frac{\lambda_{11}}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{0,12}=\frac{\lambda_{12}}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{0,13}=\frac{\lambda_{13}}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{0,14}=\frac{\lambda_{14}}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{0,15}=\frac{\lambda_{15}}{(\lambda_1+\cdots+\lambda_{16})^2}$ $m_{0,16}=\frac{\lambda_{16}}{(\lambda_1+\cdots+\lambda_{16})^2}$
$m_{10}=\frac{1}{\alpha_1}$ $m_{20}=\frac{1}{\alpha_2}$
$m_{30}=\frac{1}{\alpha_3}$ $m_{40}=\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$
$m_{41}^{(17)}=\frac{\lambda_1}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$
$m_{42}^{(18)}=\frac{\lambda_2}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$
$m_{43}^{(19)}=\frac{\lambda_3}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$
$m_{44}^{(20)}=\frac{\lambda_4}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$
$m_{44}^{(21)}=\frac{\lambda_5}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$
$m_{46}^{(22)}=\frac{\lambda_6}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4}-\frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{47}^{(23)} = \frac{\lambda_7}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{48}^{(24)} = \frac{\lambda_8}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{49}^{(25)} = \frac{\lambda_9}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,10}^{(26)} = \frac{\lambda_{10}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,11}^{(27)} = \frac{\lambda_{11}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,12}^{(28)} = \frac{\lambda_{12}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,13}^{(29)} = \frac{\lambda_{13}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,14}^{(30)} = \frac{\lambda_{14}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,15}^{(31)} = \frac{\lambda_{15}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,16}^{(32)} = \frac{\lambda_{16}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_4} - \frac{\alpha_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{4,17} = \frac{\lambda_1}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,18} = \frac{\lambda_2}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,19} = \frac{\lambda_3}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,20} = \frac{\lambda_4}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,21} = \frac{\lambda_5}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,22} = \frac{\lambda_6}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,23} = \frac{\lambda_7}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,24} = \frac{\lambda_8}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,25} = \frac{\lambda_9}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,26} = \frac{\lambda_{10}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,27} = \frac{\lambda_{11}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,28} = \frac{\lambda_{12}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,29} = \frac{\lambda_{13}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,30} = \frac{\lambda_{14}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{4,31} = \frac{\lambda_{15}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{4,32} = \frac{\lambda_{16}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{50} = \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{51}^{(33)} = \frac{\lambda_1}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{52}^{(34)} = \frac{\lambda_2}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{53}^{(35)} = \frac{\lambda_3}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{55}^{(36)} = \frac{\lambda_4}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{55}^{(37)} = \frac{\lambda_5}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{56}^{(38)} = \frac{\lambda_6}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{57}^{(39)} = \frac{\lambda_7}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{58}^{(40)} = \frac{\lambda_8}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{59}^{(41)} = \frac{\lambda_9}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,10}^{(42)} = \frac{\lambda_{10}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,11}^{(43)} = \frac{\lambda_{11}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,12}^{(44)} = \frac{\lambda_{12}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,13}^{(45)} = \frac{\lambda_{13}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,14}^{(46)} = \frac{\lambda_{14}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,15}^{(47)} = \frac{\lambda_{15}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,16}^{(48)} = \frac{\lambda_{16}}{(\lambda_1+\cdots+\lambda_{16})}\left[\frac{1}{\alpha_5} - \frac{\alpha_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}\right]$

$m_{5,33} = \frac{\lambda_1}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,34} = \frac{\lambda_2}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,35} = \frac{\lambda_3}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,36} = \frac{\lambda_4}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,37} = \frac{\lambda_5}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,38} = \frac{\lambda_6}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,39} = \frac{\lambda_7}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,40} = \frac{\lambda_8}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,41} = \frac{\lambda_9}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,42} = \frac{\lambda_{10}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,43} = \frac{\lambda_{11}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,44} = \frac{\lambda_{12}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,45} = \frac{\lambda_{13}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,46} = \frac{\lambda_{14}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{5,47} = \frac{\lambda_{15}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$ $m_{5,48} = \frac{\lambda_{16}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2}$

$m_{60} = \frac{1}{\alpha_6}$ $m_{70} = \frac{1}{\alpha_7}$

$m_{80} = \frac{1}{\alpha_8}$ $m_{90} = \frac{1}{\alpha_9}$

$m_{10,0} = \frac{1}{\alpha_{10}}$ $m_{11,0} = \frac{1}{\alpha_{11}}$

$m_{12,0} = \frac{1}{\alpha_{12}}$ $m_{13,0} = \frac{1}{\alpha_{13}}$

$m_{14,0} = \frac{1}{\alpha_{14}}$ $m_{15,0} = \frac{1}{\alpha_{15}}$

$m_{16,0} = \frac{1}{\alpha_{16}}$

The mean sojourn time (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state i, then

$\mu_i = E(T) = \Pr[T > t]\, dt = \int_0^\infty t\, Q_j(t)\, dt$

and is given by:

$\mu_0 = \frac{1}{\lambda_1+\cdots+\lambda_{16}}$; $\mu_1 = \frac{1}{\alpha_1}$; $\mu_2 = \frac{1}{\alpha_2}$; $\mu_3 = \frac{1}{\alpha_3}$;

$\mu_4 = \frac{\lambda_1+\cdots+\lambda_{16}}{(\alpha_4+\lambda_1+\cdots+\lambda_{16})^2} + \frac{1}{\alpha_4}$; $\mu_5 = \frac{\lambda_1+\cdots+\lambda_{16}}{(\alpha_5+\lambda_1+\cdots+\lambda_{16})^2} + \frac{1}{\alpha_5}$;

$\mu_6 = \frac{1}{\alpha_6}$; $\mu_7 = \frac{1}{\alpha_7}$; $\mu_8 = \frac{1}{\alpha_8}$; $\mu_9 = \frac{1}{\alpha_9}$; $\mu_{10} = \frac{1}{\alpha_{10}}$;

$\mu_{12} = \frac{1}{\alpha_{12}}$; $\mu_{11} = \frac{1}{\alpha_{11}}$; $\mu_{13} = \frac{1}{\alpha_{13}}$; $\mu_{14} = \frac{1}{\alpha_{14}}$;

$\mu_{15} = \frac{1}{\alpha_{15}}$; $\mu_{16} = \frac{1}{\alpha_{16}}$

Further,

$m_{01} + m_{02} + \cdots + m_{08} = \mu_0$; $m_{10} = \mu_1$; $m_{20} = \mu_2$; $m_{30} = \mu_3$; $m_{40} + m_{41}^{(17)} + m_{42}^{(18)} + \cdots + m_{4,16}^{(32)} + m_{4,17} + m_{4,18} + \cdots + m_{4,32} = \mu_4$; $m_{50} + m_{51}^{(33)} + m_{52}^{(34)} + \cdots + m_{5,16}^{(32)} + m_{5,33} + m_{5,34} + \cdots + m_{5,48} = \mu_5$; $m_{60} = \mu_6$; $m_{70} = \mu_7$; $m_{80} = \mu_8$; $m_{90} = \mu_9$;

$m_{10,0} = \mu_{10}$; $m_{11,0} = \mu_{11}$; $m_{12,0} = \mu_{12}$; $m_{13,0} = \mu_{13}$;

$m_{14,0} = \mu_{14}$; $m_{15,0} = \mu_{15}$; $m_{16,0} = \mu_{16}$

5 Mean time to plant failure (MTPF)

Let $\phi_i(t)$ be the $c.d.f$ of the first passage time from regenerative state i to failed state j. Regarding the failed state as absorbing, the following recursive relations are obtained:

$$\phi_0(t) = \sum_{j=1, j\neq 4,5}^{16} Q_{0,j}(t) + Q_{0,4}(t) \circledS \phi_4(t) + Q_{0,5}(t) \circledS \phi_5(t) \quad (1)$$

$$\phi_4(t) = \sum_{j=17}^{32} Q_{4,j}(t) + Q_{4,0}(t) \circledS \phi_0(t) \quad (2)$$

$$\phi_5(t) = \sum_{j=33}^{48} Q_{5,j}(t) + Q_{5,0}(t) \circledS \phi_0(t) \quad (3)$$

Taking Laplace – Stieltje's transform (L.S.T) of the equations from (1) to (3) and solving them, we get:

$$M\ T\ P = \lim_{S\to 0}\frac{1-\phi_0^{**}(s)}{S} = \frac{N}{D} \qquad (4)$$

Where N and D are as obtained.

6 Availability analysis of the system

Using the probabilistic arguments and defining $A_i(t)$ as the probability of the unit entering into upstate at instant t, giving that the unit entered in regenerative state i at $t = 0$, the following recursive relations are obtained $A_i(t)$:

$$A_0(t) = M_0(t) + \sum_{j=1}^{16} Q_{0j}(t) © A_j(t) \qquad (5)$$
$$A_1(t) = Q_{10}(t) © A_0(t) \qquad (6)$$
$$A_2(t) = Q_{20}(t) © A_0(t) \qquad (7)$$
$$A_3(t) = Q_{30}(t) © A_0(t) \qquad (8)$$
$$A_4(t) = M_4(t) + Q_{40}(t) © A_0(t) + \sum_{j=1, j\neq 5}^{16} Q_{4j}^{(1\ 4+j)}(t) © A_j(t) + Q_{44}^{(2\ 5)}(t) © A_4(t) \qquad (9)$$
$$A_5(t) = M_5(t) + Q_{50}(t) © A_0(t) + Q_{55}^{(3\ 4)}(t) © A_5(t) + \sum_{\substack{j=1\\ j\neq 4}}^{16} Q_{5j}^{(3\ 4+j)}(t) © A_j(t) \qquad (10)$$
$$A_6(t) = Q_{60}(t) © A_0(t) \qquad (11)$$
$$A_7(t) = Q_{70}(t) © A_0(t) \qquad (12)$$
$$A_8(t) = Q_{80}(t) © A_0(t) \qquad (13)$$
$$A_9(t) = Q_{90}(t) © A_0(t) \qquad (14)$$
$$A_{10}(t) = Q_{10,0}(t) © A_0(t) \qquad (15)$$
$$A_{11}(t) = Q_{11,0}(t) © A_0(t) \qquad (16)$$
$$A_{12}(t) = Q_{12,0}(t) © A_0(t) \qquad (17)$$
$$A_{13}(t) = Q_{13,0}(t) © A_0(t) \qquad (18)$$
$$A_{14}(t) = Q_{14,0}(t) © A_0(t) \qquad (19)$$
$$A_{15}(t) = Q_{15,0}(t) © A_0(t) \qquad (20)$$
$$A_{16}(t) = Q_{16,0}(t) © A_0(t) \qquad (21)$$

Taking the Laplace transforms of equations (5) to (21) and solving them for $A_0^{**}(s)$:

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0}\frac{s\ N(s)}{D(s)} = \frac{N_1}{D_1} \qquad (22)$$

where N_1 and D_1 are as obtained.

7 Busy period analysis of repairman

Using the probabilistic arguments and defining $B_0{}^*(s)$ as probability that the repairman is busy for repair at instant t, given that the unit entered in regenerative state i at $t = 0$, the following recursive relations are obtained:

$$B_0(t) = \sum_{j=1, j\neq 4,5}^{16} Q_{4,j}(t) © B_j(t) + Q_{04}(t) © B_4(t) + Q_{05}(t) © B_5(t) \qquad (23)$$
$$B_1(t) = W_1(t) + Q_{10}(t) © B_0(t) \qquad (24)$$
$$B_2(t) = W_2(t) + Q_{20}(t) © B_0(t) \qquad (25)$$
$$B_3(t) = W_3(t) + Q_{30}(t) © B_0(t) \qquad (26)$$
$$B_4(t) = W_4(t) + Q_{40}(t) © B_0(t) + \sum_{j=1, j\neq 5}^{16} Q_{4j}^{(1\ 4+j)}(t) © B_j(t) + Q_{44}^{(2\ 5)}(t) © B_4(t) \qquad (27)$$
$$B_5(t) = W_5(t) + Q_{50}(t) © B_0(t) + Q_{55}^{(3\ 4)}(t) © B_5(t) + \sum_{j=1, j\neq 4}^{16} Q_{5j}^{(3\ 4+j)}(t) © B_j(t) \qquad (28)$$
$$B_6(t) = W_6(t) + Q_{60}(t) © B_0(t) \qquad (29)$$
$$B_7(t) = W_7(t) + Q_{70}(t) © B_0(t) \qquad (30)$$
$$B_8(t) = W_8(t) + Q_{80}(t) © B_0(t) \qquad (31)$$
$$B_9(t) = W_9(t) + Q_{90}(t) © B_0(t) \qquad (32)$$
$$B_{10}(t) = W_{10}(t) + Q_{10,0}(t) © B_0(t) \qquad (33)$$
$$B_{11}(t) = W_{11}(t) + Q_{11,0}(t) © B_0(t) \qquad (34)$$
$$B_{12}(t) = W_{12}(t) + Q_{12,0}(t) © B_0(t) \qquad (35)$$
$$B_{13}(t) = W_{13}(t) + Q_{13,0}(t) © B_0(t) \qquad (36)$$
$$B_{14}(t) = W_{14}(t) + Q_{14,0}(t) © B_0(t) \qquad (37)$$
$$B_{15}(t) = W_{15}(t) + Q_{15,0}(t) © B_0(t) \qquad (38)$$
$$B_{16}(t) = W_{16}(t) + Q_{16,0}(t) © B_0(t) \qquad (39)$$

where,

$W_1(t) = \overline{G_1(t)}$; $W_2(t) = \overline{G_2(t)}$; $W_3(t) = \overline{G_3(t)}$;
$W_4(t) = \overline{G_4(t)}e^{-(\lambda_1+\cdots+\lambda_{16})t}$; $W_6(t) = \overline{G_6(t)}$;
$W_5(t) = \overline{G_5(t)}\,e^{-(\lambda_1+\cdots+\lambda_{16})t}$; $W_7(t) = \overline{G_7(t)}$;
$W_8(t) = \overline{G_8(t)}$; $W_8(t) = \overline{G_8(t)}$; $W_9(t) = \overline{G_9(t)}$;
$W_{10}(t) = \overline{G_{10}(t)}$; $W_{11}(t) = \overline{G_{11}(t)}$; $W_{12}(t) = \overline{G_{12}(t)}$;
$W_{13}(t) = \overline{G_{13}(t)}$; $W_{14}(t) = \overline{G_{14}(t)}$; $W_{15}(t) = \overline{G_{15}(t)}$;
$W_{16}(t) = \overline{G_{16}(t)}$

Now taking Laplace transforms of equations (23) to (39) and solve them, the busy period of the repairman is given by:

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_2}{D_1} \qquad (40)$$

where N_2 and D_1 are as obtained.

8 Expected number of visit by the repairman

Let $V_i(t)$ be defined as the expected number of visits for repairs in $(0, t]$, given that the plant initially starts from the regenerative state i. Using the probabilistic arguments, the following recursive relations are obtained for $V_i(t)$:

$$V_0(t) = \sum_{j=1}^{16} Q_{0j}(t)\ Ⓢ\ \left(1 + V_j(t)\right) \qquad (41)$$
$$V_1(t) = Q_{10}(t)\ Ⓢ\ V_0(t) \qquad (42)$$
$$V_2(t) = Q_{20}(t)\ Ⓢ\ V_0(t) \qquad (43)$$
$$V_3(t) = Q_{30}(t)\ Ⓢ\ V_0(t) \qquad (44)$$
$$V_4(t) = Q_{40}(t)\ Ⓢ\ V_0(t) + \sum_{j=1, j\neq 5}^{16} Q_{4j}^{(1\ 4+j)}(t)\ Ⓢ\ \left(1 + V_j(t)\right) + Q_{44}^{(2\ 5)}(s)\ Ⓢ\ (1 + V_4(t)) \qquad (45)$$
$$V_5(t) = Q_{50}(t)\ Ⓢ\ V_0(t) + Q_{55}^{(3\ 4)}(s)\ Ⓢ\ (1 + V_5(t)) + \sum_{j=1, j\neq 4}^{16} Q_{5j}^{(3\ 4+j)}(t)\ Ⓢ\ \left(1 + V_j(t)\right) \qquad (46)$$
$$V_6(t) = Q_{60}(t)\ Ⓢ\ V_0(t) \qquad (47)$$
$$V_7(t) = Q_{70}(t)\ Ⓢ\ V_0(t) \qquad (48)$$
$$V_8(t) = Q_{80}(t)\ Ⓢ\ V_0(t) \qquad (49)$$
$$V_9(t) = Q_{90}(t)\ Ⓢ\ V_0(t) \qquad (50)$$
$$V_{10}(t) = Q_{10,0}(t)\ Ⓢ\ V_0(t) \qquad (51)$$
$$V_{11}(t) = Q_{11,0}(t)\ Ⓢ\ V_0(t) \qquad (52)$$

$$V_{12}(t) = Q_{120}(t) \circledS V_0(t) \quad (53)$$
$$V_{13}(t) = Q_{130}(t) \circledS V_0(t) \quad (54)$$
$$V_{14}(t) = Q_{140}(t) \circledS V_0(t) \quad (55)$$
$$V_{15}(t) = Q_{150}(t) \circledS V_0(t) \quad (56)$$
$$V_{16}(t) = Q_{160}(t) \circledS V_0(t) \quad (57)$$

Taking Laplace Stieltje's transform of equations (41) to (57) and solving them for $V_0^{**}(s)$, the busy period of the system is given by:

$$V_0 = \lim_{s\to 0} sV_0^{**}(s) = \frac{N_3}{D_1} \quad (58)$$

Where N_3 and D_1 are as obtained.

9 Profit analysis

One of the objectives of reliability analysis is to optimize the profit incurred to the system or to the plant. Profit is defined by subtracting all expected maintenance liabilities from the total revenue. The expected total profit (P) per unit time to the plant is given by:

$$P = C_0A_0 - C_1B_0 - C_2V_0 \quad (59)$$

Where, C_0 = Revenue per unit uptime, C_1= cost per unit time for which the repairman is busy in repair and C_2= cost per visit of the repairman.

10 Particular case

For this particular case, the following have been considered:

$g_1(t)dt = \alpha_1 e^{-\alpha_1 t}$; $g_2(t)dt = \alpha_2 e^{-\alpha_2 t}$;
$g_3(t)dt = \alpha_3 e^{-\alpha_3 t}$; $g_4(t)dt = \alpha_4 e^{-\alpha_4 t}$;
$g_5(t)dt = \alpha_5 e^{-\alpha_5 t}$, $g_6(t)dt = \alpha_6 e^{-\alpha_6 t}$;
$g_7(t)dt = \alpha_7 e^{-\alpha_7 t}$; $g_8(t)dt = \alpha_8 e^{-\alpha_8 t}$;
$g_9(t)dt = \alpha_9 e^{-\alpha_9 t}$; $g_{10}(t)dt = \alpha_{10} e^{-\alpha_{10} t}$;
$g_{11}(t)dt = \alpha_{11} e^{-\alpha_{11} t}$, $g_{12}(t)dt = \alpha_{12} e^{-\alpha_{12} t}$;
$g_{13}(t)dt = \alpha_{13} e^{-\alpha_{13} t}$; $g_{14}(t)dt = \alpha_{14} e^{-\alpha_{14} t}$;
$g_{15}(t)dt = \alpha_{15} e^{-\alpha_{15} t}$; $g_{16}(t)dt = \alpha_{16} e^{-\alpha_{16} t}$.

Using the data as summarized in table I, the expressions of reliability measures as in (4), (22), (40), and (58), the following values of the plant effectiveness are obtained:

- Mean time to plant failure (MTPF) = 16.9208 hrs.
- Availability = 0.814038
- Busy period of repairman = 0.33132
- Expected number of visits = 0.07358

11 Numerical results and graphical interpretations

The above particular case has been considered for the graphical interpretation.

Figure (6.1) shows the behavior of mean time to plant failure (MTPF) with respect to failure rate (λ_1), MTPF decreases with the increase in failure rate.

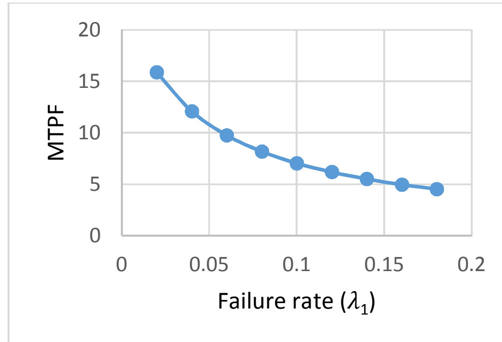

Fig. 6.1

Figure (6.2) shows the behavior of profit with respect to revenue (C_0) per unit time to different values of repair rate (α_1) it can be concluded that the profit increases with the increase in the values of C_0 and has higher values for lower rates of α_1. It can be noticed that

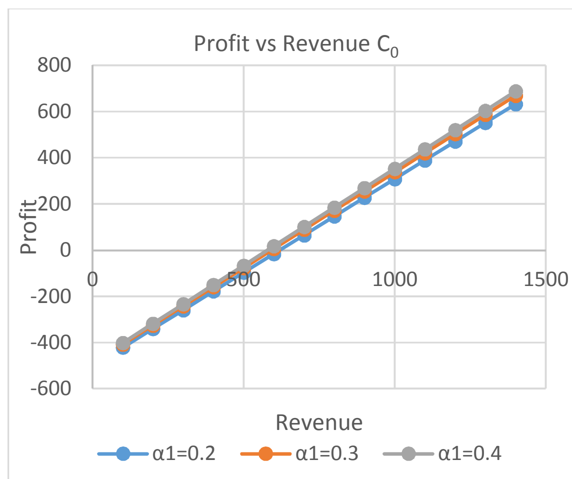

Fig. 6.2

Profit is plotted w.r.t. revenue (C_0) for different values of repair rate (α_1). It has been noted that profit is $>$ or $=$ or $<$ accordingly as C_0 is $>$ or $=$ revenue $= 600$ OMR

Figure (6.3) shows the behavior of profit (P) with respect to revenue (C_0) per unit time for different values of failure rate(λ_1). It can be concluded that the profit increases with the increase in C_0.

- For $\lambda_1 = 0.02$ and $\lambda_1 = 0.035$, the profit is positive or zero or negative according as C_0 is $>$ or $=$ or $<$ 600 OMR.

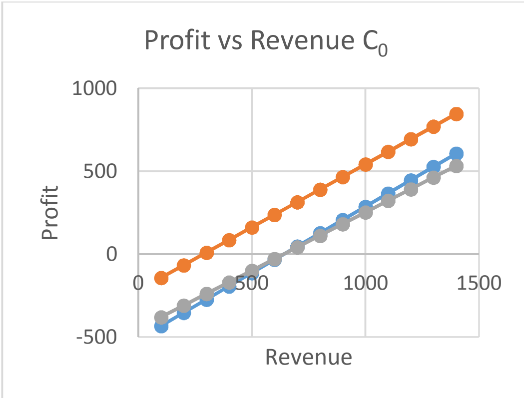

Fig. 6.3

- For $\lambda_1 = 0.03$ the profit is positive or zero or negative according as C_0 is $>$ or $=$ or $<$ or $=$ 300 OMR

Fig 6.4 demonstrates the pattern of profit with respect to revenue per unit up time (C_0) for different values of the cost of manpower for repair. The following interpretation could be achieved from this graph:

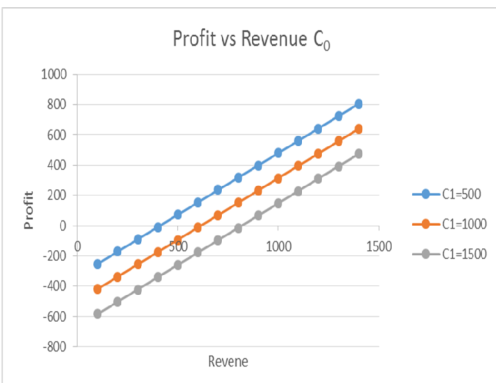

Fig. 6.4

I. For $C_1 = 500$, the profit is positive or zero or negative according as C_0> or = or < 400 OMR
II. For $C_1 = 1000$ the profit is positive or zero or negative according as C_0 > or = or < 600 OMR
III. For $C_1 = 1500$, the profit is positive or zero or negative according as C_0 > or = or < 800 OMR

12 Conclusion and future work

Reliability analysis methodology has been used to analysis a rodding anode plant in Aluminum industry where multiple unit failure is noted. Plant is operating with multiple units at eight stations. Six years of maintenance data have been used to estimate various rates and costs involved in plant maintenance. Reliability indices of interest are obtained to gauge the plant effectiveness. It has been noted that the mean time to plant failure and plant availability are on the lower side and subsequently poses question on the maintenance strategies of the company. Moreover, numerical results are used in graphs to understand the validity of the entire analysis. Mean time to plant failure verses failure rate shows a decreasing trend. Profit verses revenue for different values of repair rate, failure rate and repair cost clearly shows various cut-off points below which the optimum profitability can't be achieved. In order to improve the plant productivity multiple repair facility might help and would therefore open up a scope for further investigation.

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