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PROBABILISTIC MODELING AND ANALYSIS OF A CABLE PLANT SUBSYSTEM WITH PRIORITY TO REPAIR OVER PREVENTIVE MAINTENANCE

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Abstract – The paper presents probabilistic modeling and analysis of a cable plant subsystem with two identical machines operating in parallel. Seven years failure data of the cable plant depicts three types of maintenance for the subsystem: repair, minor preventive maintenance and major preventive maintenance. The subsystem undergoes repair upon failure while minor and major preventive maintenance is performed as per schedule. Priority is given to repair over preventive maintenance. Mean time to failure, availability, expected busy period and expected number of repairs have been estimated by analyzing the subsystem using semi Markov process and regenerative point technique. Graphs have been established to demonstrate simulated results.

Keywords – cable plant, failure, repair, minor/major scheduled preventive maintenance, probabilistic modeling, semi Markov process, regenerative process.

NOTATIONS
MIPM Minor preventive maintenance
MAPM Major preventive maintenance
PM Preventive maintenance
Sᵢ State i
βᵢ Estimated value of rate of requirement of MIPM
βⱼ Estimated value of rate of requirement of MAPM
λ Estimated value of failure rate
f₁(t) Probability density function of MIPM times
f₂(t) Probability density function of MAPM times
g(t) Probability density function of repair times
α₁ Estimated value of rate of performing MIPM
α₂ Estimated value of rate of performing MAPM
γ Estimated value of repair rate
Qᵢ Cumulative distribution function from Sᵢ to Sⱼ
qᵢ Probability density function from Sᵢ to Sⱼ

© Laplace convolution
S Laplace Stieljes’s convolution
* Laplace transform
** Laplace Stieljes’s transform

I. INTRODUCTION

Literature shows ample work reported by researchers in the area of reliability specifically using stochastic or probabilistic analysis for industrial systems under different operating conditions and assumptions. Probabilistic analysis using modeling approach plays an important role in understanding the system behaviour in terms of reliability indices and cost benefit evaluation. Gulshan et al. [1] analyzed system with perfect repair under partial failure mode and priority for repair to completely failed unit, Gopalan & Bhanu [2] considered two unit repairable system subject to online preventive maintenance and/or repair, Tuteja et al. [3]-[5] worked for two units system with regular repairman who is not always available, system with perfect repair at partial failure or complete failure mode, and the profit evaluation of a two units cold standby system with tiredness and two types of repairmen. Rizwan et al. [6]-[12] analyzed cold and hot standby systems with single unit and two units under different failure and repair situations where the reliability indices of interest are obtained and the cost benefit analysis of the systems are carried out. Mathew et al. [13]-[19] extensively analyzed the continuous casting plant and studied the variations under different operating conditions of the plant. Detailed analysis was reported for desalination plant by Padmavathi et al. [20] with online repair under emergency shutdowns, Rizwan et al. [21] with repair/maintenance strategy on first come first served basis, Padmavathi et al. [22]-[26] continued on desalination plant with priority for repair over maintenance, comparative analysis between the plant models, analysis under major and minor
failures consideration, analysis by prioritizing repair over maintenance under major/minor failures, and comparative analysis between the plant models portraying two operating conditions of the plant as to which model is better than the other. The methodology was further extended for various industrial systems analysis by Gupta & Gupta [27] with post inspection concept, Ram et al. [28] waiting repair strategy, Malhotra & Taneja [29] both units operative on demand, Niwas et al. [30] obtained mean time to system failure and profit of a single unit system with inspection for feasibility of repair beyond warranty. Later, Rizwan et al. [31]-[33] focused on waste water treatment plant and anaerobic batch reactor and reliability indices of interest were obtained in order to assess the plant/reactor performance. Sharma & Kaur [34] presented cost benefit analysis of a compressor standby system with preference of service, repair and replacement is given to recently failed unit. Recently, Naithani et al. [35] discussed probabilistic analysis of a 3-unit induced draft fan system with one warm standby with priority to repair of the unit in working state. Taj et al. [36]-[37] further explored the methodology to analyze two different subsystems of a cable plant and obtained reliability indices of interest. Hence, the methodology for system analysis has been widely studied. However, the novelty lies in its application to a different industrial situation. Electric cables being widely used in the construction industry, therefore, the analysis of cable manufacturing plant is of great importance from reliability perspective. Subsystems being instrumental in the entire plant effectiveness, need to be analysed separately before addressing the entire plant.

Thus, the paper is an attempt to present analysis of a cable plant subsystem using seven years maintenance data of a plant currently operational in Oman. Based on various operating states of the subsystem, a detailed analysis is carried out using semi-Markov process and regenerative point technique. Outcome of the entire analysis is measured in terms of mean time to subsystem failure, availability of the subsystem, expected busy period of the repairman and expected number of subsystem repairs. Simulated results are demonstrated graphically.

II. DESCRIPTION OF THE SUBSYSTEM

The subsystem under consideration consists of two identical machines operating in parallel. Seven years maintenance data of the cable plant depicts three types of maintenance practices for the subsystem: repair, minor preventive maintenance (MIPM) and major preventive maintenance (MAPM). The subsystem is repaired upon normal failures whereas MIPM and MAPM are carried out at scheduled basis. Priority is given to repair over preventive maintenance (PM).

Possible transition states of the subsystem are described below:

State 0 (S₀): both machines are operative
State 1 (S₁): one machine is down for MIPM, other machine is operative
State 2 (S₂): one machine is down for MAPM, other machine is operative
State 3 (S₃): one machine is under repair, other machine is operative
State 4 (S₄): one machine is under repair, other machine is waiting for MAPM
State 5 (S₅): one machine is under repair, other machine is waiting for repair

S₀, S₁, S₂, S₃ and S₄ are regenerative states whereas S₅ is a non-regenerative state. Table 1 gives the rates of transition from S_i to S_j, 0 stands for no transition to the mentioned state. Failure rates are taken as exponential whereas repair/PM rates as arbitrary.

<table>
<thead>
<tr>
<th>S_i</th>
<th>S₀</th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
<th>S₄</th>
<th>S₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₀</td>
<td>0</td>
<td>β₁</td>
<td>β₂</td>
<td>2λ</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₁</td>
<td>f₁(t)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₂</td>
<td>f₂(t)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>λ</td>
<td>0</td>
</tr>
<tr>
<td>S₃</td>
<td>g(t)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>λ</td>
</tr>
<tr>
<td>S₄</td>
<td>0</td>
<td>g(t)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₅</td>
<td>0</td>
<td>0</td>
<td>g(t)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

For non-regenerative state 5
S₂ to S₃ via S₄

\[
d\Omega_{53}^S(t) = (\lambda e^{-\lambda t} + 1)g(t)dt
\]

Table 1: Rates for the subsystem
Table 2 shows estimated values of rates of repair/failure and rates of performing/requirement of PM.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Rate/ hour</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \beta_1 ), rate of requirement of MIPM</td>
<td>0.0018</td>
</tr>
<tr>
<td>2</td>
<td>( \beta_2 ), rate of requirement of MAPM</td>
<td>0.0005</td>
</tr>
<tr>
<td>3</td>
<td>( \lambda ), failure rate</td>
<td>0.0054</td>
</tr>
<tr>
<td>4</td>
<td>( \alpha_1 ), rate of performing MIPM</td>
<td>0.8668</td>
</tr>
<tr>
<td>5</td>
<td>( \alpha_2 ), rate of performing MAPM</td>
<td>0.0510</td>
</tr>
<tr>
<td>6</td>
<td>( \gamma ), repair rate</td>
<td>0.1936</td>
</tr>
</tbody>
</table>

Table 2: Estimated values of rates for the subsystem

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Transition probabilities from \( S_i \) to \( S_j \) are given by the equations (1) – (10):

\[
\begin{align*}
dQ_{01}(t) &= \beta_1 e^{-(\beta_1 + \beta_2 + 2\lambda)t} dt \\
dQ_{02}(t) &= \beta_2 e^{-(\beta_1 + \beta_2 + 2\lambda)t} dt \\
dQ_{03}(t) &= 2\lambda e^{-(\beta_1 + \beta_2 + 2\lambda)t} dt \\
dQ_{10}(t) &= f_1(t) dt \\
dQ_{20}(t) &= e^{-\lambda t} f_2(t) dt \\
dQ_{24}(t) &= \lambda e^{-\lambda t} g_2(t) dt \\
dQ_{30}(t) &= e^{-\lambda t} g(t) dt \\
dQ_{35}(t) &= \lambda e^{-\lambda t} G(t) dt \\
dQ_{33}(t) &= (\lambda e^{-\lambda t} g(t)) dt \\
dQ_{42}(t) &= g(t) dt
\end{align*}
\]

The non zero elements \( p_{ij} \) can be obtained as follows:

\[
\begin{align*}
p_{01} &= \frac{\beta_1}{\beta_1 + \beta_2 + 2\lambda} \\
p_{02} &= \frac{\beta_2}{\beta_1 + \beta_2 + 2\lambda} \\
p_{03} &= \frac{2\lambda}{\beta_1 + \beta_2 + 2\lambda} \\
p_{10} &= 1 \\
p_{20} &= f_2^*(\lambda) \\
p_{24} &= 1 - f_2^*(\lambda) \\
p_{30} &= g^*(\lambda) \\
p_{35} &= 1 - g^*(\lambda) \\
p_{33} &= 1 - g^*(\lambda) \\
p_{42} &= 1
\end{align*}
\]

Following relations can easily be verified

\[
\begin{align*}
p_{01} + p_{02} + p_{03} &= 1 \\
p_{10} &= 1 \\
p_{20} + p_{24} &= 1 \\
p_{30} + p_{35} &= 1 \\
p_{30} + p_{33} &= 1 \\
p_{42} &= 1
\end{align*}
\]
The mean sojourn time $\mu_i$ in the regenerative state $i$ is defined as the time of stay in that state before transition to any other state. If $T$ denotes the sojourn time in the regenerative state $i$, then

$$\mu_i = E(T) = \int_0^{\infty} Pr[T > t] \, dt$$

$$\mu_0 = \frac{1}{\beta_1 + \beta_2 + 2\lambda}$$

$$\mu_1 = \int_0^{\infty} F_1(t) \, dt$$

$$\mu_2 = \int_0^{\infty} e^{-\lambda t} F_2(t) \, dt$$

$$\mu_3 = \int_0^{\infty} e^{-\lambda t} G(t) \, dt$$

$$\mu_4 = \int_0^{\infty} e^{-\lambda t} \int G(t) \, dt$$

**IV. MATHEMATICAL ANALYSIS**

**A. Mean time to subsystem failure**

Let $\phi_i(t)$ be the cumulative distribution function of the first passage time from regenerative state $i$ to a failed state $j$. Using probabilistic arguments, the following recursive relations for $\phi_i(t)$ are obtained:

$$\phi_0(t) = Q_{01}(t) S_{01}(t) + Q_{02}(t) S_{02}(t) + Q_{03}(t) S_{03}(t)$$

$$\phi_1(t) = Q_{10}(t) S_{10}(t)$$

$$\phi_2(t) = Q_{20}(t) S_{20}(t) + Q_{24}(t) S_{24}(t)$$

$$\phi_3(t) = Q_{30}(t) S_{30}(t) + Q_{35}(t) S_{35}(t)$$

$$\phi_4(t) = Q_{42}(t) S_{42}(t)$$

Taking Laplace Stieltjes transform of equations (32-36) and solving for $\phi_0^{**}(s)$, we get equation (37)

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

Mean time to subsystem failure when the subsystem started at the beginning of state 0 is given by equation (38)

$$\text{Mean time to subsystem failure} = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = P_{02} \mu_2 + P_{02} P_{24} \mu_4 + P_{20} \mu_0 + P_{20} P_{01} \mu_1 + P_{20} P_{03} \mu_3$$

$$D = P_{20} P_{03} P_{35}$$

**B. Availability of the subsystem**

Using probabilistic arguments of pointwise availability and defining $A_i(t)$ as the probability that the plant is in up state at instant $t$, given that it enters the regenerative state $i$ at $t = 0$, the following recursive relations are obtained:

$$A_0(t) = M_0(t) + Q_{01}(t) A_1(t) + Q_{02}(t) A_2(t) + Q_{03}(t) A_3(t)$$

$$A_1(t) = q_{10}(t) A_0(t)$$

$$A_2(t) = q_{20}(t) A_0(t) + Q_{24}(t) A_4(t)$$

$$A_3(t) = q_{30}(t) A_0(t) + Q_{35}(t) A_5(t)$$

$$A_4(t) = q_{42}(t) A_2(t)$$

Taking Laplace transform of equations (39-43) and solving for $A_0^{**}(s)$, we get equation (44)

$$A_0^{**}(s) = \frac{N_1(s)}{D_1(s)}$$

In steady state, availability of the subsystem is given by equation (45)

$$A_0 = \lim_{s \to 0} s A_0^{**}(s) = \frac{N_1}{D_1}$$

where

$$N_1 = P_{20} P_{30} \mu_0$$

$$D_1 = P_{20} P_{30} \mu_0 + P_{30} P_{02} \mu_2 + P_{03} P_{20} \mu_4 + P_{20} P_{30} P_{01} \mu_1 + P_{30} P_{02} P_{24} \mu_4$$
C. Expected busy period of the repairman

Using probabilistic arguments and defining $B_i(t)$ as the probability that the repairman is busy at instant $t$, given that the system entered regenerative state $i$ at $t = 0$, we get the following recursive relations:

\[ B_0(t) = q_{01}(t) \ast B_1(t) + q_{02}(t) \ast B_2(t) \ast q_{03}(t) \ast B_3(t) \]  \hspace{1cm} (46)

\[ B_1(t) = q_{10}(t) \ast B_0(t) \]  \hspace{1cm} (47)

\[ B_2(t) = q_{20}(t) \ast B_0(t) + q_{24}(t) \ast B_4(t) \]  \hspace{1cm} (48)

\[ B_3(t) = W_3(t) + q_{30}(t) \ast B_0(t) + q_{34}(t) \ast B_4(t) \]  \hspace{1cm} (49)

\[ B_4(t) = W_4(t) + q_{42}(t) \ast B_2(t) \]  \hspace{1cm} (50)

Here, $W_3(t) = e^{-\lambda t} G(t)$ and $W_4(t) = \overline{G(t)}$

Taking Laplace Stieltjes transform of equations (46-50) and solving for $B_0^*(s)$, we obtain equation (51)

\[ B_0^*(s) = \frac{N_2(s)}{D_1(s)} \]  \hspace{1cm} (51)

In steady state, expected busy period of the repairman is given by equation (52)

\[ B_0 = \lim_{s \to 0} s B_0^*(s) = \frac{N_2}{D_1} \]  \hspace{1cm} (52)

where

\[ N_2 = p_{24} p_{03} \mu_3 + p_{30} p_{02} p_{24} \mu_4 \]

$D_1$ is already specified

D. Expected number of subsystem repairs

Using probabilistic arguments and defining $R_i(t)$ as the expected number of repairs in $(0, t]$, given that the subsystem entered regenerative state $i$ at $t = 0$, we get the following recursive relations:

\[ R_0(t) = Q_{01}(t) \ast R_1(t) + Q_{02}(t) \ast R_2(t) + Q_{03}(t) \ast (R_3(t) + 1) \]  \hspace{1cm} (53)

\[ R_1(t) = Q_{10}(t) \ast R_0(t) \]  \hspace{1cm} (54)

\[ R_2(t) = Q_{20}(t) \ast R_0(t) + Q_{24}(t) \ast (R_4(t) + 1) \]  \hspace{1cm} (55)

\[ R_3(t) = Q_{30}(t) \ast R_0(t) + Q_{34}(t) \ast (R_4(t) + 1) \]  \hspace{1cm} (56)

\[ R_4(t) = Q_{42}(t) \ast R_2(t) \]  \hspace{1cm} (57)

Taking Laplace Stieltjes transform of equations (53-57) and solving for $R_0^{**}(s)$, we get equation (58)

\[ R_0^{**}(s) = \frac{N_3(s)}{D_1(s)} \]  \hspace{1cm} (58)

In steady state, expected number of repairs per unit time is given by equation (59)

\[ R_0 = \lim_{s \to 0} s R_0^{**}(s) = \frac{N_3}{D_1} \]  \hspace{1cm} (59)

where

\[ N_3 = p_{24} p_{03} + p_{30} p_{02} p_{24} \]

$D_1$ is already specified

V. PARTICULAR CASE

For this particular case, the failure times are exponential, whereas other times follow arbitrary distribution.

\[ f_1(t) = \alpha_1 e^{-\alpha_1 t} \]  \hspace{1cm} (60)

\[ f_2(t) = \alpha_2 e^{-\alpha_2 t} \]  \hspace{1cm} (61)

\[ g(t) = ye^{-yt} \]  \hspace{1cm} (62)

Using the estimated values from table 2 and expressions (38), (45), (52), (59); the following reliability indices are obtained:

Mean time to subsystem failure = 3644.3713 hours

Availability of the subsystem = 0.9350

Expected busy period of the repairman = 0.0079

Expected number of subsystem repairs = 0.0104
VI. GRAPHICAL INTERPRETATION

Figures 1 and 2 show the trend of mean time to subsystem failure and availability of the subsystem respectively when plotted against failure rate $\lambda$. It can be seen that mean time to subsystem failure and availability of the subsystem decreases with the increase in failure rate $\lambda$.

Figure 1

<table>
<thead>
<tr>
<th>mean time to failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>4000</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2

<table>
<thead>
<tr>
<th>availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

Indices for a cable plant subsystem are obtained to measure the subsystem effectiveness in terms of mean time to subsystem failure, availability of the subsystem, expected busy period of the repairman and expected number of subsystem repairs. Simulated results are shown graphically. There is potential scope of extending system analysis for multiple machines operating in parallel/series with various online/offline maintenance arrangements.
References


