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Nistazakis, Hector E.; Stassinakis, A.N.; Sinanovic, Sinan; Popoola, Wasiu O.; Tombras, George S.

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Performance of quadrature amplitude modulation orthogonal frequency division multiplexing-based free space optical links with non-linear clipping effect over gamma–gamma modelled turbulence channels

H.E. Nistazakis[1], A.N. Stassinakis[1], S. Sinanović[2], W.O. Popoola[3,*] and G.S. Tombras

[1]: Department of Electronics, Computers, Telecommunications and Control, Faculty of Physics,
National and Kapodistrian University of Athens, Athens, 15784, Greece
e-mails: {enistaz; a-stasinakis; gtombras}@phys.uoa.gr

[2]: Department of Engineering, School of Engineering and Built Environment,
Glasgow Caledonian University, Glasgow, United Kingdom
e-mail: sinan.sinanovic@gcu.ac.uk

3: Institute for Digital Communications,
School of Engineering, University of Edinburgh,
Edinburgh, EH9 3JL, UK
e-mail: w.popoola@ed.ac.uk

*: Corresponding author, W.O. Popoola, w.popoola@ed.ac.uk

Abstract

The free space optical (FSO) communication systems have attracted significant research and commercial interest in the last few years due to their low installation and operational cost along with their very high performance characteristics. However, for terrestrial FSO links the optical signal propagates through the atmosphere which exhibits time varying behavior that implies variations of links' performance. In this work, we estimate the performance metrics for terrestrial FSO links which are using the orthogonal frequency division multiplexing (OFDM) technique with a quadrature amplitude modulation (QAM) scheme over turbulence channels. More specifically, we investigate the influence of the nonlinear clipping effect of the OFDM scheme,

along with the atmospheric turbulence modeled using the gamma-gamma distribution. Both effects significantly influence the performance of the link and here we derive closed form mathematical expressions for the estimation of the average signal to noise ratio (SNR), the outage probability and the average bit error rate (BER) that vital for FSO system performance characterization. Finally, using these expressions, we present the corresponding numerical results for common parameter values of the FSO links and we investigate the accuracy of our expressions for marginal cases with nearly negligible turbulence effect.

I. Introduction

Free space optical FSO communication systems have attracted significant research and commercial interest in the last few years because they can achieve very high performance characteristics with relatively low installation and operational cost [1]-[11]. The main disadvantage of the FSO links is caused by the fact that the optical signal propagates through the atmosphere which shows varying characteristics that affect their performance and reliability significantly. An important phenomenon that impairs the system's performance is the atmospheric turbulence induced scintillation effect [1]-[3], [9], [11]-[14]. Due to this effect, the irradiance of the optical signal at the receiver's side does not remain invariable; it fluctuates rapidly with respect to time and space. In order to model this irradiance fluctuation, many statistical distributions have been used, depending on the atmospheric turbulence strength [12]-[24]. One model which has been proven to accurately predict weak to strong atmospheric turbulence conditions is the gamma-gamma turbulence model [10], [25]-[30].

A multiplexing scheme commonly studied in optical communication systems is the OFDM. It offers the capability to transmit the data in parallel subcarriers that are orthogonal to each other.This technique is an effective way of achieving multicarrier data transmission using multiple relatively narrow band subcarriers. Each subcarrier is modulated by the information signal using an advanced modulation scheme such as the QAM scheme [31]-[38]. Due to the subcarriers' orthogonality, the intersymbol interference (ISI) is minimal and the different subcarriers can thus be separated relatively easily at the receiver [9], [10].

Beside the influence of the propagation channel in optical communication systems, signal distortions/clippings can occur due to other various reasons [31], [34].These include limited dynamic range and nonlinearities in the optical front-ends. Furthermore, with optical-OFDM (O-OFDM), the signal can have very large peak-to-average power ratio (PAPR) [33]-[36] resulting in further clipping at the optical front-end. O-OFDM follows Gaussian distribution due to the fact that many signals are added together so that central limit theorem can be applied. In fact, it has been demonstrated that even for as low as 64 carriers the Gaussian distribution assumption holds [34]. The Gaussianity of the non-distorted signal means that Bussgang theorem can be applied to analyze the clipping [34] and other nonlinear distortion [31] effects. More specifically, clipping of the signal can be represented with an attenuated data signal and additional uncorrelated non-Gaussian noise. The clipping-induced attenuation and the additional noise power can be computed by applying Bussgang theorem as reported in [31].

A number of statistical models now exist to model atmospheric turbulence including gamma-gamma, gamma, I-K, K, negative exponential and log-normal distributions. Based on these models, the impact of atmospheric turbulence on FSO systems has been studied in literature and mathematical expressions derived to estimate the system performance [12]-[30]. The need to further increase the transmission rate and mitigate inter-symbol interference (ISI) has contributed to the study of OFDM for FSO [31]-[38]. The effect of atmospheric turbulence on such OFDM-FSO links has been studied and reported in [10], [33], [36]-[38]. However, OFDM has innate challenges that significantly affect its performance. These include clipping noise and susceptibility to nonlinearity effects; the limitations imposed on the OFDM FSO system by these have been reported in [31], [34], [35].

Although the individual impairments (turbulence and clipping/nonlinearity noise) have all been studied in isolation, it is desirable to have a unified analytical framework that combines these effects and accurately evaluate the system performance. This void in OFDM FSO research is addressed in this paper. Specifically, we present accurate closed form analytical expressions for evaluating the performance of OFDM FSO in the presence of turbulence and clipping/nonlinearity noise. Due to their significance in the design of a communication system, the performance evaluations metrics considered in this work are average SNR, average BER and outage probability.

The remainder of this work is organized as follows: in section II, we present the system and the channel model with the gamma-gamma distribution, in section III we present the average SNR at the receiver, in section IV we derive the expression for the outage probability of the link and in section V, we present the average BER of the QAM OFDM wireless optical link with clipping over gamma-gamma modelled turbulence channels. Numerical results for various clipping levels and turbulence strengths are presented in section VI while section VII contains the concluding remarks.

II. System and Channel Model

In terrestrial optical wireless communication links, the atmospheric turbulence phenomenon causes the scintillation effect which results in irradiance fluctuations at the receiver. Depending on the strength of the fluctuation, a number of statistical distributions exist to model the scintillation effect. Most of these models are very accurate for specific atmospheric turbulence conditions. The gamma-gamma turbulence model considered in this work takes into account both the small and the large scale contributions to scintillation effect [13], [14]. Moreover, it has been proven that this model is suitably accurate for weak to strong turbulence and its parameters can be directly estimated from the link's characteristics [1]-[2],[10], [13],[14], [27], [28].

The probability density function (PDF) of the gamma-gamma distribution, $f_I(I)$, as a function of the normalized irradiance I is given as [13]:

$$f_I(I)=\frac{(ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)}I^{\frac{a+b}{2}-1}K_{a-b}\left(2\sqrt{abI}\right) \tag{1}$$

with $K_{a-b}(.)$ being the modified Bessel function of the second kind of order $(a-b)$, $\Gamma(.)$ is the gamma function and the parameters a and b can be estimated from the link's characteristics through the following expressions [13], [14], [27], [28]:

$$a=\left[\exp\left(\frac{0.49\delta^2}{\left(1+0.18d^2+0.56\delta^{12/5}\right)^{7/6}}\right)-1\right]^{-1}$$

$$b=\left[\exp\left(\frac{0.51\delta^2\left(1+0.69\delta^{12/5}\right)^{-5/6}}{\left(1+0.9d^2+0.62d^2\delta^{12/5}\right)^{5/6}}\right)-1\right]^{-1} \tag{2}$$

where $d=\sqrt{kD^2/(4L)}$, $k=2\pi/\lambda$, is the optical wave number, $\lambda$is the operational wavelength of the communication system, L represents the length of the link while D stands for the receiver's aperture diameter. The parameter δ^2 stands for the Rytov variance which for relatively weak turbulence conditions is given as $\delta^2=1.23C_n^2k^{7/6}L^{11/6}$ [13]. C_n^2 is the refractive index structure parameter which depends on the altitude and the atmospheric conditions and is given as $C_n^2 = \left(79 \times 10^{-6} P / T^2\right)^2 C_T^2$, where P, T and C_T^2 stand for the atmospheric pressure, temperature and temperature structure constant respectively [27], [39]. Moreover, it is worth mentioning that the value of the parameter C_n^2 varies from 10^{-17} to 10^{-13} $\mathrm{m}^{-2/3}$ for weak to strong turbulence conditions [29].

Taking into account the irradiance fluctuations at the receiver's side due to the scintillation effect, the optical channel can be considered as a fading one. Moreover, if the OFDM technique is used, it is assumed as flat fading one over the entire OFDM bandwidth of up to 20 MHz [34], [38]. Additionally, O-OFDM systems suffer from signal clipping due to the limited dynamic range of the optical end and nonlinearities. Considering the clipping effect, the optical path gain coefficient is given as [34], [39], [41]:

$$\gamma = \frac{S_{PD}\rho_{PD}GI}{E[P_t]\sqrt{r_l}} \qquad (3)$$

with $S_{PD} = \pi D^2/4$ and ρ_{PD} stand for the photosensitive area and the responsivity of the photodetector (PD), respectively, while G represents the gain of the transimpedance amplifier, $E[P_t]$ is the average transmitted optical power and r_l, the resistance over which the current of the receiver is measured [34], [41], [42].

From (3) and [34, eq. 25], the following mathematical expression is derived for the estimation of the instantaneous effective electrical SNR per bit, $\mu(I)$,in the OFDM based FSO link as a function of the undistorted electrical SNR per bit, γ_b [34]:

$$\mu(I) = \frac{K^2 \Lambda^2 G_{DC} P_B \gamma_b I^2}{\sigma_{clip}^2 \Lambda^2 G_{DC} G_B \gamma_b I^2 + G_B P_B} \quad (4)$$

where $\Lambda = S_{PD} \rho_{PD} G / \left(E[P_t]\sqrt{r_l}\right)$, K quantifies the effective attenuation factor due to nonlinear clipping, G_{DC} denotes the attenuation of the useful electrical signal power of time domain electrical signal due to the biasing of the transmitter front-end in the least signal clipping scenario, $G_B$is the utilization factor for the double-sided bandwidth B of the OFDM frame, P_B represents an average electrical power per bit and σ^2_{clip} denotes the clipping noise variance [34].

III. Estimation of the Average SNR

In this section we estimate the average electrical SNR arriving at the receiver which is a significant metric for the system's performance evaluation. For this, we are taking into account both the clipping noise of the QAM-O-OFDM signal as well as the atmospheric turbulence induced fading. Thus, the average electrical SNR per bit, $\overline{\mu}$, is estimated by integrating the instantaneous electrical SNR per bit at the receiver over the PDF of the gamma-gamma turbulence model. The average electrical SNR per bit is thus given by:

$$\overline{\mu} = \int_0^\infty \mu(I) f_I(I) dI \qquad (5)$$

By combining (1), (4) and (5) we derive the average SNR at the receiver, which includes both clipping and turbulence effects as:

$$\bar{\mu} = \frac{\Xi(ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)} \int_0^{\infty} \frac{I^{\frac{a+b}{2}+1}}{\Psi I^2 + \Omega} K_{a-b}\left(2\sqrt{abI}\right) dI \qquad (6)$$

with $\Xi = K^2 \Lambda^2 G_{DC} P_B \gamma_b$, $\Psi = \sigma_{clip}^2 \Lambda^2 G_{DC} G_B \gamma_b$ and $\Omega = G_B P_B$.

By substituting the expression $\left(\Psi I^2/\Omega + 1\right)^{-1}$ and the Bessel function with their appropriate hypergeometric functions and more specifically with the Meijer ones [43], i.e. $\left(\Psi I^2/\Omega + 1\right)^{-1} = G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega}\middle|\begin{matrix}0\\0\end{matrix}\right)$ and $K_{a-b}\left(2\sqrt{abI}\right) = \frac{1}{2} G_{0,2}^{2,0}\left(abI\middle|\begin{matrix}- & -\\ \frac{a-b}{2}, & \frac{b-a}{2}\end{matrix}\right)$, we obtain the following closed form mathematical expression for the estimation of the average effective electrical SNR of the OFDM-based FSO system with the clipping and the atmospheric turbulence effect:

$$\bar{\mu} = \frac{2^{a+b+1}\Xi}{\pi\Omega(ab)^2\Gamma(a)\Gamma(b)} G_{5,1}^{1,5}\left(\frac{16\Psi}{\Omega(ab)^2}\middle|\begin{matrix}0, -\frac{a+1}{2}, -\frac{a}{2}, -\frac{b+1}{2}, -\frac{b}{2}\\ 0\end{matrix}\right) \qquad (7)$$

where $G_{p,q}^{m,n}[\cdot]$ stands for the Meijer-function which is a standard built in function which can be evaluated with most of the well-known mathematical software packages [27], [43].

IV. Link's Outage Probability

Another significant metric estimating the availability of an FSO link is the outage probability, P_{out}. Here, this quantity is estimated as a function of the threshold SNR, μ_{th}. This threshold SNR represents the minimum SNR required for satisfactory performance in the presence of turbulence induced fading and nonlinear clipping. The outage probability gives the probability that the received SNR falls below this critical value and thus, the optical link becomes unavailable. In general, the outage probability of a communication link is given as [23]:

$$P_{out}\left(\mu_{th}\right)=\Pr\left(\mu\leq\mu_{th}\right)=F_{\mu}\left(\mu_{th}\right) \tag{8}$$

where $F_{\mu}\left(\mu\right)$ stands for the cumulative density function (CDF) of the appropriate distribution model (here, we are using the gamma-gamma distribution), as a function of the instantaneous SNR per bit, μ, at the receiver. Thus, by integrating the PDF given by (1) and using expression (4), we arrive at the following CDF:

$$F_{\mu}\left(\mu\right)=\frac{\left(ab\right)^{\frac{a+b}{2}}\left(\frac{\Omega\mu}{\Xi-\Psi\mu}\right)^{\frac{a+b}{4}}}{\Gamma\left(a\right)\Gamma\left(b\right)}G_{1,3}^{2,1}\left(ab\sqrt{\frac{\Omega\mu}{\Xi-\Psi\mu}}\left|\begin{matrix}1-\frac{a+b}{2}\\ \frac{a-b}{2},\frac{b-a}{2},-\frac{a+b}{2}\end{matrix}\right.\right) \tag{9}$$

Next, by substituting (9) in (8),we derive the following closed form mathematical expression for the outage probability of the OFDM based FSO link with clipping effect and gamma-gamma modelled atmospheric turbulence as:

$$P_{out}\left(\mu_{th}\right)=\frac{\left(\frac{\Omega}{\xi\gamma_{b}/\mu_{th}-\zeta\gamma_{b}}\right)^{\frac{a+b}{4}}}{\Gamma\left(a\right)\Gamma\left(b\right)\left(ab\right)^{-\frac{a+b}{2}}}G_{1,3}^{2,1}\left(ab\sqrt{\frac{\Omega}{\xi\gamma_{b}/\mu_{th}-\zeta\gamma_{b}}}\left|\begin{matrix}1-\frac{a+b}{2}\\ \frac{a-b}{2},\frac{b-a}{2},-\frac{a+b}{2}\end{matrix}\right.\right) \tag{10}$$

where $\xi = \Xi \gamma_b^{-1}$ and $\zeta = \Psi \gamma_b^{-1}$. Thus, the outage probability, $P_{out}\left(\mu_{th}\right)$, can now be estimated as a function of the undistorted electrical SNR per bit,γ_b, for any value of threshold SNR at the receiver.

V. Average BER

The BER metric of the M-QAM OFDM FSO link can be accurately estimated, as a function of the instantaneous irradiance I at the receiver's side, through the following expression [34], [44]:

$$BER(I) = \lambda_1 Q\left(\lambda_2 \sqrt{\mu(I)}\right) + \lambda_3 Q\left(3\lambda_2 \sqrt{\mu(I)}\right) \qquad (11)$$

where $\lambda_1 = \frac{4\left(\sqrt{M}-1\right)}{\sqrt{M}\log_2(M)}$, $\lambda_2 = \sqrt{\frac{3\log_2(M)}{M-1}}$, $\lambda_3 = \frac{4\left(\sqrt{M}-2\right)}{\sqrt{M}\log_2(M)}$, M stands for the QAM modulation order and *Q(.)*represents the *Q*-function. By combining the *Q*-function defined as $Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$ with $erfc(x) \approx \frac{1}{6}\exp\left(-x^2\right) + \frac{1}{2}\exp\left(-\frac{4x^2}{3}\right)$ [43], and (11), we derive the following expression for the BER:

$$\begin{aligned} BER(I) \approx & \frac{\lambda_1}{4}\left\{\frac{1}{3}\exp\left(-\frac{\lambda_2^2 \mu(I)}{2}\right) + \exp\left(-\frac{2\lambda_2^2 \mu(I)}{3}\right)\right\} + \\ & + \frac{\lambda_3}{4}\left\{\frac{1}{3}\exp\left(-\frac{9\lambda_2^2 \mu(I)}{2}\right) + \exp\left(-6\lambda_2^2 \mu(I)\right)\right\} \end{aligned} \qquad (12)$$

Equation (12), estimates the instantaneous BER as a function of the instantaneous SNR and the average BER (ABER) is estimated using the corresponding PDF through the following integral [4], [10], [24], [37]:

$$ABER = \int_0^{\infty} BER(I)\, f_I(I)\, dI \qquad (13)$$

By substituting (1) and (12) in (13), the ABER becomes:

$$\begin{aligned} ABER \approx {} & \kappa_1 \int_0^{\infty} \exp\left(\frac{\Omega\mu_1}{\Psi I^2 + \Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_2 \int_0^{\infty} \exp\left(\frac{\Omega\mu_2}{\Psi I^2 + \Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \\ & + \kappa_3 \int_0^{\infty} \exp\left(\frac{\Omega\mu_3}{\Psi I^2 + \Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_4 \int_0^{\infty} \exp\left(\frac{\Omega\mu_3}{\Psi I^2 + \Omega}\right) I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI \end{aligned} \qquad (14)$$

with $\kappa_1 = (\lambda_1\lambda_4/12)\exp(-\mu_1)$, $\mu_1 = \lambda_2^2\Xi/2\Psi$, $\kappa_2 = (\lambda_1\lambda_4/4)\exp(-\mu_2)$, $\mu_2 = 2\lambda_2^2\Xi/3\Psi$, $\kappa_3 = (\lambda_3\lambda_4/12)\exp(-\mu_3)$, $\mu_3 = 9\lambda_2^2\Xi/2\Psi$, $\kappa_4 = (\lambda_3\lambda_4/4)\exp(-\mu_4)$, $\mu_4 = 6\lambda_2^2\Xi/\Psi$, $\lambda_4 = 2(ab)^{\frac{a+b}{2}} \big/ \left(\Gamma(a)\Gamma(b)\right)$, $\lambda_5 = (a+b-2)/2$.

Next, by substituting the exponential function with the infinite summation, i.e. $\exp(x) = \sum_{n=0}^{+\infty} \frac{x^n}{n!}$, and the quantity $\left(\Psi I^2/\Omega + 1\right)^{-n}$ with the corresponding Meijer function [43], i.e. $\left(\Psi I^2/\Omega + 1\right)^{-n} = \frac{1}{(n-1)!} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix}\right)$ for $n \geq 1$, expression (14) now takes the following form:

$$
\begin{aligned}
ABER \approx\ & \kappa_1 \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_1 \sum_{n=1}^{+\infty} \frac{n\mu_1^n}{(n!)^2} \int_0^\infty I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix} \right) K_{a-b}\left(2\sqrt{abI}\right) dI + \\
& + \kappa_2 \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_2 \sum_{n=1}^{+\infty} \frac{n\mu_2^n}{(n!)^2} \int_0^\infty I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix} \right) K_{a-b}\left(2\sqrt{abI}\right) dI + \\
& + \kappa_3 \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_3 \sum_{n=1}^{+\infty} \frac{n\mu_3^n}{(n!)^2} \int_0^\infty I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix} \right) K_{a-b}\left(2\sqrt{abI}\right) dI + \\
& + \kappa_4 \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \kappa_4 \sum_{n=1}^{+\infty} \frac{n\mu_4^n}{(n!)^2} \int_0^\infty I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix} \right) K_{a-b}\left(2\sqrt{abI}\right) dI
\end{aligned}
\tag{15}
$$

This result can be further simplified to the following expression:

$$
\begin{aligned}
ABER \approx\ & \sum_{i=1}^{4} \kappa_i \int_0^\infty I^{\lambda_5} K_{a-b}\left(2\sqrt{abI}\right) dI + \\
& + \sum_{n=1}^{+\infty} \left\{ \frac{n \sum_{i=1}^{4} \left(\kappa_i \mu_i^n\right)}{(n!)^2} \int_0^{+\infty} I^{\lambda_5} G_{1,1}^{1,1}\left(\frac{\Psi I^2}{\Omega} \middle| \begin{matrix} 1-n \\ 0 \end{matrix} \right) K_{a-b}\left(2\sqrt{abI}\right) dI \right\}
\end{aligned}
\tag{16}
$$

By evaluating the integrals involved in (16), we obtain the following expression (17) that accurately predicts the average BER of the *M*-QAM OFDM based FSO link in the presence of gamma-gamma turbulence and signal clipping.

$$
\begin{aligned}
ABER \approx\ & \lambda_4^{-1} \sum_{i=1}^{4} \kappa_i + \\
& + \sum_{n=1}^{\infty} \left\{ \frac{2^{a+b-3} n \sum_{i=1}^{4} \left(\kappa_i \mu_i^n\right)}{\pi (ab)^{\frac{a+b}{2}} (n!)^2} G_{5,1}^{1,5}\left(\frac{16\Psi}{\Omega (ab)^2} \middle| \begin{matrix} 1-n, \frac{1-a}{2}, \frac{2-a}{2}, \frac{1-b}{2}, \frac{2-b}{2} \\ 0 \end{matrix} \right) \right\}
\end{aligned}
\tag{17}
$$

Although expression (17) contains an infinite number of summands for the index *n*, it does in reality match the result of the numerical integration of (14) even for small values of *n* (i.e. for *n* up to5). This will be further discussed in the numerical results section that follows.

VI. Numerical Results

From the derived mathematical expressions (7), (10) and (17), we can now estimate the average electrical SNR per bit $\overline{\mu}$, for the OFDM-based FSO, the outage probability and the average BER of the link with clipping noise and turbulence modelled with the gamma-gamma distribution. More specifically, we have chosen the value of C_n^2 as 2×10^{-14} m$^{-2/3}$ and 2×10^{-13} m$^{-2/3}$to represent moderate and strong turbulence conditions, respectively, link length *L*, as 1000m and operational wavelength λ=0.83 μm. Additionally, for the σ_{clip}^2 we have chosen the value 0.002, ρ_{PD}=0.5, G_{TIA}=50000, r_{load}=50Ω, $E[x_{time}(k)]$=10, G_B=0.5, P_B=0.5, G_{DC}=0.6, and *K*=0.5 [34], [42]. Also we used two values for the S_{PD}, i.e. 0.002 m^2 and 0.005 m^2 and three different QAM levels, *M* =4, 16 and 64.

A parameter which can be used in order to estimate the influence of the atmospheric turbulence effect at FSO system's performance is the Fried parameter, r_0 [47]. The value of the fraction D/r_0, constitutes an important quantity which shows how strongly the turbulence affects the link's performance. Thus, for a terrestrial FSO link with nearly horizontal propagation path, where the parameter C_n^2 can be assumed to remain invariable for the whole link length, the Fried parameter is given as [47], [48]:

$$r_0 = 3.02\left(Lk^2 C_n^2\right)^{-3/5} \qquad (18)$$

By substituting the parameters mentioned in the above paragraph into (18), for the fraction D/r_0 we obtain the values 1.14 and 4.55 for S_{pd} equal to 0.002 m^2 and 0.005 m^2, respectively, and weak atmospheric turbulence conditions, i.e. C_n^2=2×10^{-14} m$^{-2/3}$, while for the case of strong turbulence, i.e. C_n^2=2×10^{-13} m$^{-2/3}$, the corresponding values are 1.81 and 7.2. These outcomes show that the atmospheric turbulence strength and the aperture diameter of the receiver affect significantly the system's performance even for relatively short link lengths. This conclusion will be verified further from the results that follow.

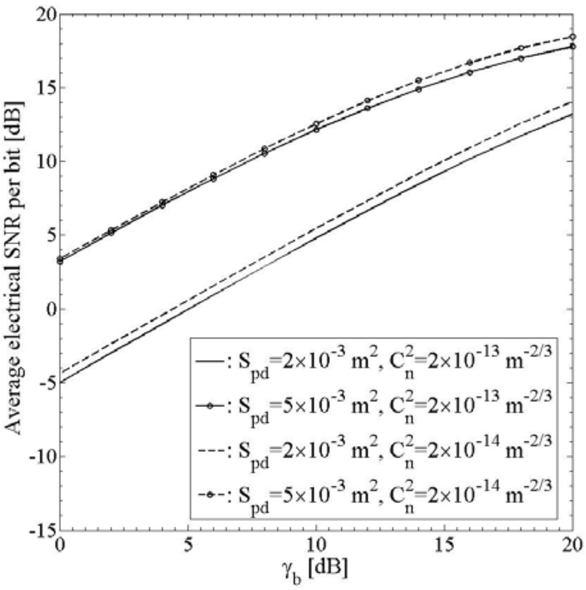

Figure 1: Average electrical SNR per bit, $\overline{\mu}$, as a function of γ_b, taking into account the signal clipping effect, under the action of moderate or strong turbulence conditions.

In Figure 1, we present the average electrical SNR per bit for the OFDM-based turbulent FSO link with clipping, as obtained in (7). As it is shown here, the average electrical SNR arriving at the PD depends strongly on the atmospheric turbulence conditions and the strength of the signal clipping effect.

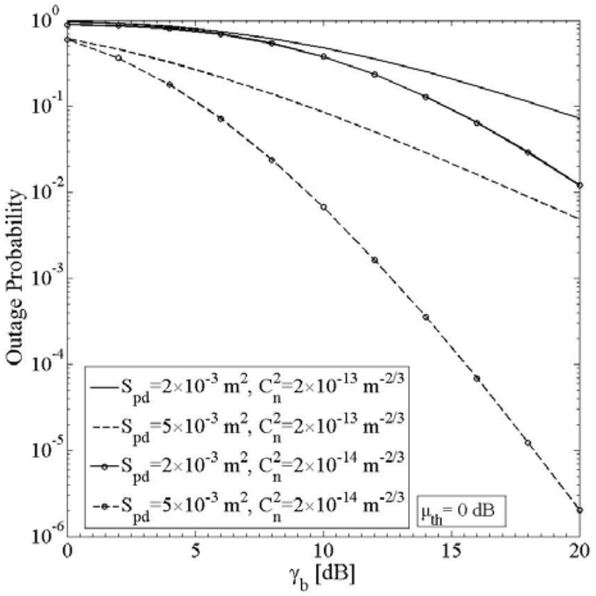

Figure 2: Outage probability, P_{out}, for μ_{th}=0dB, as a function of γ_b, for moderate and strong turbulence conditions.

Next, in Figures 2 and 3, we present the outage probability of the wireless optical link with clipping and gamma-gamma turbulence effects, as a function of γ_b, for two values of the μ_{th}, namely, 0 dB and 2 dB.These figures show the influence of the receiver's surface area and the strength of the atmospheric turbulence effect as very significant parameters for the system's availability. Thus, if the S_{PD} is large and the turbulence weak, the outage probability becomes very small values even for relatively low values of the undistorted electrical SNR per bit.

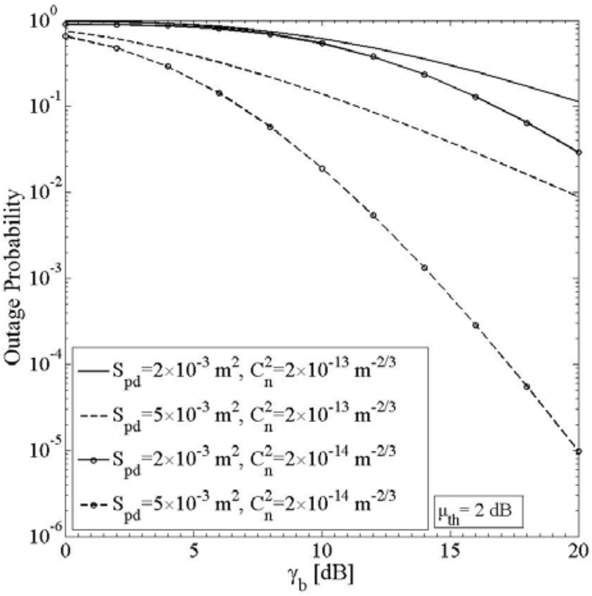

Figure 3: Outage probability, P_{out}, for μ_{th}=2dB, as a function of γ_b, for moderate and strong turbulence conditions.

In Figures 4 and 5 we present the average BER as a function of the undistorted electrical SNR per bit for moderate to strong turbulence, various values of M and S_{PD}. As in the previous case, the influence of the S_{PD} and C_n^2 parameters is very significant, especially for M= 4 or 16.

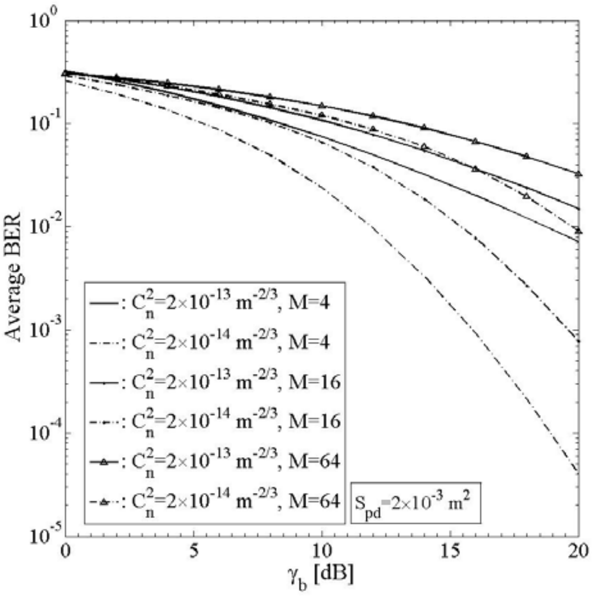

Figure 4: Average BER, as a function of γ_b, for moderate and strong turbulence conditions, for M=4, 16 or 64 and S_{PD}=0.002 m^2.

As mentioned in the previous section, the expression (17), which gives the average BER, includes a summation of infinite number of terms. Thus, a significant issue is to estimate the number of terms which have to be taken into account, at the evaluation of the average BER using (17) with the lower error. In order to calculate this estimation error, the integral of (14) has been evaluated numerically and the result of the expression (17) for n=5 as well, and the relative estimation error of the estimation of the average BER using the expressions (14) and (17), E_R, has been calculated for various parameters values. The results show that even for this small value of n the relative estimation error did not exceed 0.05%.

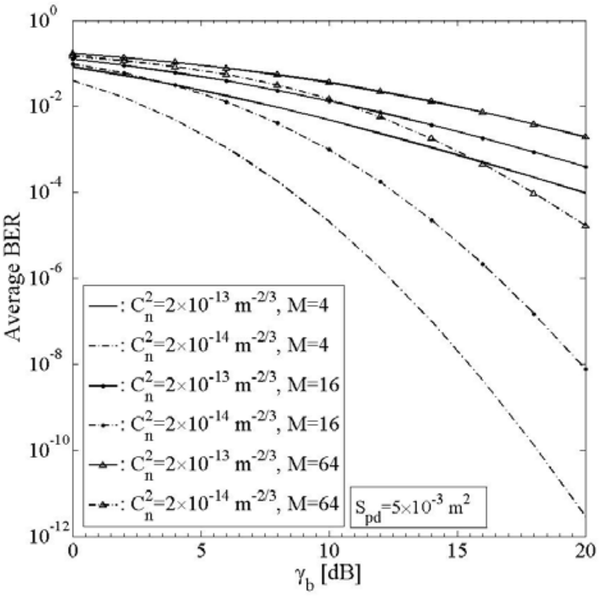

Figure 5: Average BER, as a function of γ_b, for moderate and strong turbulence conditions, for M=4, 16 or 64 and S_{PD}=0.005 m^2.

More specifically, in Figure 6, we present the absolute value of the relative estimation error of the average BER versus the average electrical SNR for the above mentioned fixed parameters and S_{pd}=0.002 m^2, M=4, while the refractive index structure parameter, C_n^2, takes the values 2×10^{-14} m$^{-2/3}$ and 2×10^{-13} m$^{-2/3}$. It is clear that the relative error can be assumed to be negligible for most common practical cases. Moreover, this relative estimation error can be further reduced by increasing the value of *n* accordingly.

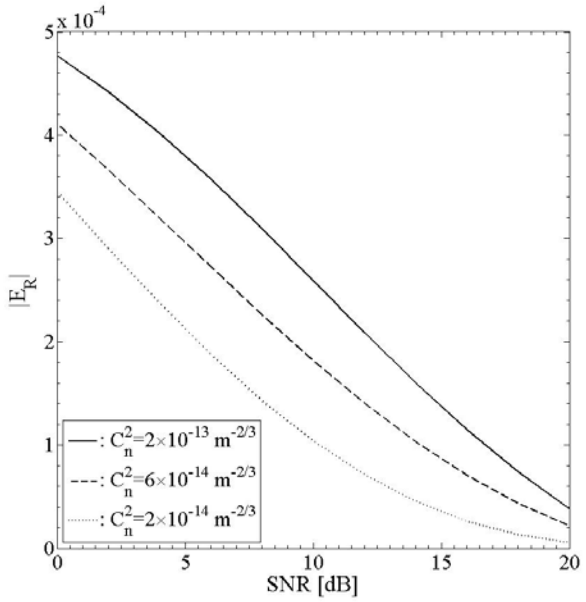

Figure 6: Absolute value of the relative estimation error, $|E_R|$, of the average BER, for various parameter values, as a function of the undistorted electrical SNR per bit.

Next, in Figure 7, we present results for average BER evaluated from expression (17) for the case of extremely weak atmospheric turbulence; i.e. very small values of $C_n^2$and the corresponding ones obtained from (12) for the case without turbulence at all. More specifically we present the average BER,for the fixed parameter values mentioned above, S_{pd}= 0.002 m^2, M=4, and C_n^2=5×10^{-13} m$^{-2/3}$, 5×10^{-14} m$^{-2/3}$ and 5×10^{-15} m$^{-2/3}$. The results shown here, verify numerically the fact that the smaller values of the refractive index structure parameter correspond to very weak influence of the atmospheric turbulence effect and the average BER curve tends to coincide with the limiting case without turbulence, i.e. the results obtained from equation (12).

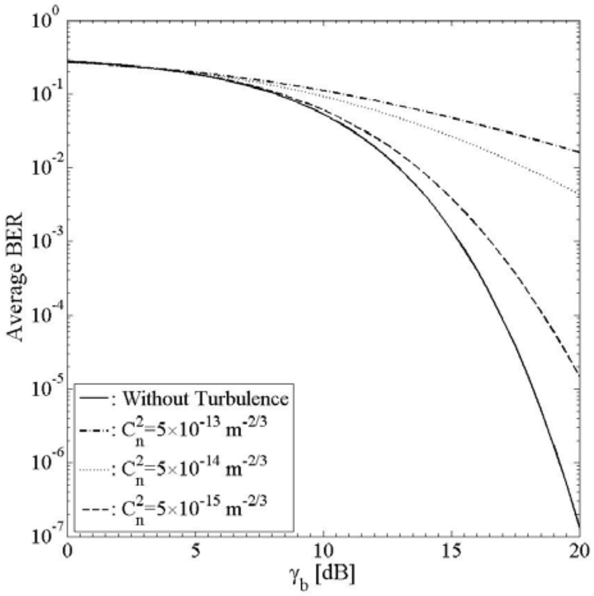

Figure 7: Average BER, as a function of γ_b, for various cases of turbulence strength and in the limiting case with clipping noise and turbulence's absence.

The results that we presented above have been obtained using realistic values for link's parameters. Thus, the obtained outcomes show the dependence of the performance of the specific optical wireless communication system from the strength of the clipping noise and the atmospheric turbulence effect. However, taking into account the large number of parameters which affect significantly the system's operation and have been mentioned above, the presented numerical results can be used, mainly, as a qualitatively measure for the dependence of system's performance on each specific factor of the link. Consequently, the derived close mathematical expressions, i.e. (7), (10) and (17), can be used for the estimation of performance results for any parameter value needed for the designing and implementation of any particular QAM OFDM FSO links with clipping noise over atmospheric turbulence channels.

VII. Conclusions

We have derived analytically closed form mathematical expressions for the key performance metrics for an OFDM FSO channel which explicitly take into account the turbulence modelled with the gamma-gamma distribution and nonlinear distortions caused by the clipping effect. In particular, we have derived expressions for the average SNR, average BER and outage probability, as a function of the physical parameters of the OFDM FSO link. Finally we presented numerical simulation results in order to show the dependence of these key metrics on turbulence, constellation size and the PD's active area which can be used for design of OFDM FSO links.

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