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Bee-foraging learning particle swarm optimization

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Abstract

Numerous particle swarm optimization (PSO) algorithms have been developed for solving numerical optimization problems in recent years. However, most of existing PSO algorithms have only one search phase. There is no strengthened search phase for the well-performed particles, and also no re-initialization phase for the exhausted particles. These issues may still restrict the performance of PSO for complex optimization problems. In this paper, inspired by the bee-foraging search mechanism of artificial bee colony algorithm, a novel bee-foraging learning PSO (BFL-PSO) algorithm is proposed. Different from existing PSO algorithms, the proposed BFL-PSO has three different search phases, namely employed learning, onlooker learning and scout learning. The employed learning phase works like traditional one-phase-based PSO, while the onlooker learning phase performs strengthened search around those well-performed particles to exploit promising solutions, and the scout learning phase re-initializes those exhausted particles to introduce new diversity. The proposed BFL-PSO is comprehensively evaluated on CEC2014 benchmark functions, and compared with state-of-the-art PSO algorithms as well as artificial bee colony algorithms. The experimental results show that BFL-PSO achieves very competitive performance in terms of solution accuracy. In addition, the effectiveness of the newly introduced onlooker learning and scout learning phases in BFL-PSO is verified.

Keywords: Particle swarm optimization, bee-foraging learning mechanism, artificial bee colony, numerical optimization

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1. Introduction

Particle swarm optimization (PSO) is a swarm intelligence algorithm originally proposed by Kennedy and Eberhart, inspired by the social behavior of bird swarm in search of destination. In PSO, each particle has its velocity and position, and also memorizes its own best position found in history. By using the information from the personal best and global best positions, each particle adjusts its velocity and position to search for optimal solution [1]. PSO has attracted much attention in the past two decades due to its easy implementation, conceptual simplicity and rapid convergence, and has also been successfully applied for solving optimization problems in many areas [2, 3, 4, 5].

PSO shows promising performance in many optimization problems, however, it often suffers from premature convergence and stagnation in the search process. To enhance the performance, many improved PSO variants have been proposed, such as CLPSO (comprehensive learning PSO) [6], OLPSO (Orthogonal learning PSO) [7], SLPSO (social learning PSO) [8], SRPSO (self-regulating PSO) [9], EPSO (ensemble PSO) [10], BLPSO (biogeography-based learning PSO) [11], GLPSO (genetic learning PSO) [12], FMPSO (fitness-based multi-role PSO) [13], to name a few.

Although these PSO variants alleviate the drawbacks of the PSO to a certain extent, most of them do not include strengthened search strategy for those particles that perform well during the searching process. On the other hand, these particles which have not improved their fitness over a certain number of iterations may have suffered from stagnation, and no strategy is used to handle this situation. These issues may still restrict the performance of PSO for solving complex global optimization problems (GOPs).

Artificial bee colony (ABC) is another swarm intelligence algorithm inspired by the bee foraging behavior [14]. ABC has division-of-labor feature that is different from the other swarm-based meta-heuristics including PSO and ant colony optimization [15]. Specifically, the optimization process of ABC simulates three different groups of bees, that is, employed bees, onlookers, and scouts, searching for food sources [16]. Employed bees search around the food sources to seek for better ones, onlooker bees perform strengthened search around those food sources with better fitness, and scout bees perform random search when the exhausted food sources are identified.

In this paper, by integrating the bee-foraging search mechanism into PSO, we develop a novel bee-foraging learning PSO (BFL-PSO) algorithm. Different from existing PSO algorithms, BFL-PSO involves three kinds of search phases, namely employed learning, onlooker learning and

scout learning. The employed learning plays a role as the traditional single-phase-based PSO, while the onlooker learning and scout learning are two new search phases which do not exist in traditional PSO. The onlooker learning performs strengthened search around those particles with better fitness for exploitation, while the scout learning re-initializes the velocities and positions of those exhausted particles for exploration. The proposed BFL-PSO algorithm is evaluated on CEC2014 benchmark functions [17] in comparison with the state-of-the-art PSO algorithms as well as ABC algorithms to verify its performance.

The remainder of this paper is organized as follows. Section 2 presents the literature review. Section 3 proposes a bee-foraging learning PSO (BFL-PSO) algorithm. Section 4 provides the experimental results and comparisons. Section 5 applies the BFL-PSO algorithm to solve economic power dispatch problems. Finally, Section 6 concludes this paper.

2. Literature review

Numerous PSO algorithms have been developed in the past two decades. In this section, we provide a brief review on the recently well-established PSO algorithms. For a more comprehensive review of past PSO, please refer to [18] and [19].

2.1. PSO with parameter control mechanisms

PSO has control parameters including inertia weight, acceleration coefficients and population size, and properly setting these parameters may largely improve PSO's performance. In [9], inspired from the human cognitive psychology, a self-regulating PSO (SRPSO) algorithm was developed, which employed a self-regulating inertia weight for the best particle to improve the exploration. In [13], a fitness-based multi-role PSO (FMPSO) algorithm was proposed with multi-role-based parameter mechanism. Based on the fitness values, the particles in FMPSO play three different roles (leader, Rambler and follower), and different inertia weights and acceleration coefficients are set for different roles, which helps the population perform different search patterns. In [20], according to its distance to personal best position, each particle uses different inertia weights in different dimensions. Moreover, the acceleration coefficients are calculated adaptively according to a stability condition. In [21], a hybrid PSO with sine cosine acceleration coefficients (HPSOSCAC) was proposed, in which sine cosine based acceleration coefficients efficiently controls the local search and convergence to global optimal solution. In [22], a population size adaptive mechanism was designed for PSO, in which two operators namely deleting particles and adding particles are performed during the evolutionary process. When the fitness landscape

is easy to optimize, some bad particles are deleted, while when the population cannot find more promising solutions, some pleasurable particles are added to the population.

2.2. PSO with population topology or multi-swarm

Population topology and multi-swarm techniques can help enhance the population diversity, which may help improve PSO's performance for complex multimodal problems. In [23], the ring topology was introduced into genetic learning PSO, and it is shown that replacing the global topology with ring topology for exemplar generation can enhance the diversity and exploratory of genetic learning PSO. In [24], the cellular structured population was employed for quantum-inspired PSO (QPSO), and the cQPSO algorithm was proposed. In cQPSO, the particles are distributed in a two-dimensional grid and only interact with their neighbors. In [25], a heterogeneous comprehensive learning PSO (HCLPSO) using two subpopulations was proposed. In HCLPSO, one subpopulation focuses on exploration, while the other focuses on exploitation. In [26], a multi-population cooperative PSO with a mixed mutation strategy (MPCPSO) was proposed. In MPCPSO, the population is divided into elitist sub-population and general sub-population based on the fitness values. Elitist sub-population and general sub-population adopt different learning strategies, with the objectives of guiding particle evolution and maintaining population diversity, respectively.

2.3. PSO with new learning strategies

Designing new learning strategies can utilize the search information more efficiently, and it is another promising way to develop new PSO algorithms. In [8], a social learning mechanism was introduced into PSO to develop a social learning PSO (SLPSO) algorithm. In the social learning mechanism, each particle learns from different better particles in the current swarm, and a dimension-based parameter setting method is used to enhance the algorithmic robustness to the search dimension. In [12], a genetic learning strategy was devised for PSO, which uses genetic evolution operators to breed promising exemplars based on the personal best information. Using the GLS, the global search ability and search efficiency of genetic learning PSO (GLPSO) are both improved. In [27], inspired by interactive learning behaviors taken place among different human groups, a new PSO algorithm with interswarm interactive learning strategy (IILPSO) was proposed. In IILPSO, the particles are divided into two groups, when the best particle's fitness values of both sub-groups do not improve, the interswarm interactive learning behavior is triggered. In [10], an ensemble learning method was devised and an ensemble PSO (ESPO)

algorithm was proposed. In EPSO, based on a self-adaptive mechanism, five different updating strategies are employed to improve the versatility and robustness of PSO for different optimization problems. In [11], a biogeography-based learning PSO (BLPSO) algorithm was proposed, in which each particle updates its position by using all particles' personal best positions through the biogeography-based migration mechanism. In [28], a dimensional learning strategy was devised for PSO, which is used to integrate and discover the promising information among the global best position and the personal best positions of all particles. In [29], a level-based learning mechanism was devised and a level-based learning swarm optimizer (LLSO) was proposed for large-scale global optimization. In LLSO, the particles are divided into several levels according to their fitness values, and each particle randomly select two predominant particles from two different higher levels to guide the learning.

2.4. PSO hybrid with non-ABC meta-heuristics

Many hybrid PSO algorithms have been developed, since hybrid PSO with other meta-heuristic algorithms can combine their merits and overcome drawbacks. In [30], teaching-learning-based optimization (TLBO) was introduced into PSO, and a bidirectional teaching and peer-learning PSO (BTPLPSO) algorithm was proposed. In BTPLPSO, the particles that fail to improve their fitness will enter into a peer-learning phase, and learn from the high-quality exemplars. In [31], cultural evolution algorithm was introduced into quantum-behaved PSO (QPSO), and a cultural quantum-behaved PSO (CMOQPSO) was proposed. In CMOQPSO, a belief space containing three types of knowledge is extracted from the particle swarm and also used to generate global best positions for the particles. In [32], a hybrid PSO algorithm with levy flight (PSOLF) was proposed. In PSOLF, the levy flight mechanism from cuckoo search algorithm is used for updating the particles' velocities, and it is shown that PSOLF has high convergence rate. In [33], by hybridizing PSO with harmony search, a hybrid algorithm called HHSPSO-GDS was proposed. In HHSPSO-GDS, the improvisation mechanism of harmony search and a global dimension selection are incorporated into the algorithm which effectively enhances the convergence. In [34], a hybrid crisscross search PSO (CSPSO) algorithm was proposed, in which the population is updated through crisscross search and PSO in sequence. In [35], a differential mutation strategy was employed to update the three best particles, and a differential mutation based social learning PSO (DSPSO) was proposed. DSPSO overcomes the shortcoming of unsatisfactory search efficiency in social learning PSO algorithm.

2.5. PSO hybrid with artificial bee colony

The hybridization of PSO and artificial bee colony (ABC) has also attracted researchers' attention, and several hybrid PSO-ABC algorithms have been proposed. Firstly, several hybridizations combine PSO and ABC in a dual-swarm parallel framework. For example, in [36], an integrated algorithm called IABAP was proposed by parallelly executing PSO and ABC algorithms and exchanging the information of two sub-swarms. In [37], a hybrid algorithm called HPA was proposed, where a recombination operator is performed on the best solutions of PSO and ABC, and the obtained solutions are fed back to PSO and ABC. Secondly, several hybridizations combine PSO and ABC in a cascade framework. In [38], an adaptive bee and particle swarm optimization algorithm called ABeePSO was proposed, in which the ABC algorithm is employed to generate the swarm diversity when the PSO algorithm stagnates. In [39], two search phases from ABC and one local search phase from PSO were combined, and a hybrid algorithm called PS-ABC was proposed. Thirdly, several hybridizations use ABC's search strategy in PSO or use PSO's search strategy in ABC. In [40], by using the ABC's search strategy on the personal best positions of the standard PSO, a hybrid ABC-SPSO algorithm was proposed. In [41], by using PSO's search equation in bee search phase as well as chaotic search operation on the global best solution, a hybrid algorithm called ABC-PS was proposed. In [42], the concept of velocity was introduced into ABC, and velocity based ABC (VABC) algorithm was developed. In [43], the global best solution and an elitist archive were used to modify the search equation of ABC, and particle swarm inspired multi-elitist ABC (PS-MEABC) algorithm was developed. In [44], local best and global best solutions were used to modify the search equations of employed bees and onlooker bees, and a lbest-gbest ABC (LGABC) algorithm was developed.

This paper will integrate the bee-foraging search mechanism into PSO, and developed a novel bee-foraging learning PSO (BFL-PSO) algorithm. Therefore, our BFL-PSO algorithm presents a new way that combines PSO and ABC algorithms.

3. Proposed bee-foraging learning PSO (BFL-PSO) algorithm

3.1. Bee-foraging learning model

PSO uses a population of particles searching for good solutions. In PSO, each particle has its own position and velocity, and updates itself by learning from the personal best positions and global best position. On the other hand, ABC employs three kinds of bees (i.e., employed, onlooker and scout) searching for good food sources, and different bees are responsible for different

tasks [45]. Inspired by the division-of-labor of ABC, this paper proposes a new learning model for PSO, called bee-foraging learning (BFL) model.

In the BFL model, the population firstly initializes N particles; each has its own position $\mathbf{x}_i$, velocity $\mathbf{v}_i$ and personal best position $\mathbf{pbest}_i$. Then, it will enter into three learning phases, called employed learning, onlooker learning, and scout learning, as shown in Fig.1.

Figure 1: Bee-foraging learning model

3.1.1. Employed learning

In the employed learning phase, the particles work like the employed bees in ABC. Specifically, each particle updates its own velocity $\mathbf{v}_i$ and position $\mathbf{x}_i$ by learning from the personal best positions (i.e., $\mathbf{pbest}_i$) and global best position (i.e., $\mathbf{gbest}$). The new positions are updated as follow:

$$\begin{cases} \mathbf{v}_i^{new} = \mu(\mathbf{v}_i^{old}, \mathbf{x}_i^{old}, \mathbf{pbest}_i, \mathbf{gbest}) \\ \mathbf{x}_i^{new} = \mathbf{x}_i^{old} + \mathbf{v}_i^{new} \end{cases} \quad (1)$$

where $\mathbf{v}_i^{old}$ and $\mathbf{x}_i^{old}$ are the velocity and position of the i -th particle in previous iteration; $\mathbf{v}_i^{new}$ and $\mathbf{x}_i^{new}$ are the new velocity and position of the i -th particle in current iteration; and function $\mu(\cdot)$ denotes the velocity updating method in PSO. If $\mathbf{x}_i^{new}$ is better than $\mathbf{pbest}_i$ (i.e., $f(\mathbf{x}_i^{new}) < f(\mathbf{pbest}_i)$, where $f(\cdot)$ is the objective function for a minimization problem), then

171 $\mathbf{pbest}_i$ is replaced by $\mathbf{x}_i^{new}$.

172 At the same time, for those particles who fail to update their personal best position $\mathbf{pbest}_i$,
 173 their counter will be increased, i.e., $count(i) = count(i) + 1$, while for those particles who succeed
 174 in finding better positions, their counter will be reset, i.e., $count(i) = 0$.

175 3.1.2. Onlooker learning

176 In the onlooker learning phase, those particles with better fitness are selected to perform
 177 strengthened search, which is like the onlooker bees in ABC. The fitness of each particle is firstly
 178 calculated based on its $f(\mathbf{pbest}_i)$ as follows:

$$fit(\mathbf{x}_i) = \begin{cases} \frac{1}{1+f(\mathbf{pbest}_i)} & \text{if } f(\mathbf{pbest}_i) \geq 0 \\ 1 + |f(\mathbf{pbest}_i)| & \text{otherwise} \end{cases} \quad (2)$$

179 and the probability p_i for selecting the i -th particle is calculated as follows:

$$p_i = \frac{fit(\mathbf{x}_i)}{\sum_{i=1}^N fit(\mathbf{x}_i)} \quad (3)$$

180 Now, particles are selected based on the probability p_i using the roulette method. Those
 181 particles with better $\mathbf{pbest}_i$ have larger probability to be selected. Assume the s -th particle $\mathbf{x}_s$
 182 is selected, it will use Eq.(1) to generate a new position $\mathbf{x}_s^{new}$. If $\mathbf{x}_s^{new}$ is better than $\mathbf{pbest}_s$,
 183 then $\mathbf{pbest}_s$ is replaced by $\mathbf{x}_s^{new}$.

184 At the same time, for those particles who fail to update their personal best position $\mathbf{pbest}_s$,
 185 their counter will be increased, i.e., $count(s) = count(s) + 1$, and for those particles who succeed
 186 in finding better positions, their counter will be reset, i.e., $count(s) = 0$.

187 3.1.3. Scout learning

188 In the scout learning phase, those particles which fail to update their personal best position
 189 $\mathbf{pbest}_i$ in a certain number of iterations (i.e., $count(i) \geq limit$) are deemed as exhausted. The
 190 particle will be abandoned, and its position $\mathbf{x}_i$, velocity $\mathbf{v}_i$ and personal best position $\mathbf{pbest}_i$ are
 191 all re-initialized randomly in the search space.

192 **Remark 1:** The BFL model incorporates the ABC's division-of-labor feature into PSO. Tradi-
 193 tional PSO often has only one learning phase, which is like the employed learning in the BFL
 194 model. By contrast, the BFL model introduces two new search phases, i.e., onlooker learning
 195 and scout learning. The onlooker learning performs strengthened search around those positions

196 with better fitness, which can further enhance the performance of PSO by exploiting promis-
 197 ing solutions. The scout learning re-generates the velocities and positions of those exhausted
 198 particles, which can bring new diversity when the exhausted particles suffer from stagnation.

199 3.2. Description of BLF-PSO Algorithm

200 A novel bee-foraging learning PSO (BFL-PSO) algorithm is established based on the BFL
 201 model. The velocity and position updating equations of biogeography-based learning PSO (BLP-
 202 SO) [11] are employed in the BFL-PSO algorithm. The velocity and position updating equations
 203 in BFL-PSO are as follows:

$$\begin{cases} \mathbf{v}_i^{new} = w \cdot \mathbf{v}_i^{old} + c \cdot \mathbf{rand} \cdot (\mathbf{pbest}_{\tau_i} - \mathbf{x}_i^{old}) \\ \mathbf{x}_i^{new} = \mathbf{x}_i^{old} + \mathbf{v}_i^{new} \end{cases} \quad (4)$$

204 where w is the inertia weight; c is the learning factor; $\mathbf{rand}$ is a random vector uniformly dis-
 205 tributed within $[0, 1]^D$; $\tau_i = [\tau_i(1), \dots, \tau_i(d), \dots, \tau_i(D)]$ is the index vector of learning exemplar
 206 for the i -th particle, which is generated by biogeography-based exemplar generation method as
 207 described in **Appendix 1**.

208 The pseudocode of the BFL-PSO algorithm ¹ is presented in **Algorithm 1**. In **Algorithm**
 209 **1**, lines 5~15 is the pseudocode of employed learning phase, which is the same as the traditional
 210 one-phase-based BLPSO. Lines 16~29 is the pseudocode of onlooker learning phase, and lines
 211 30~36 is the pseudocode of scout learning phase.

¹In BFL-PSO, the learning exemplar indices τ_i for the particles are re-generated when there is a stagnation for G continuous iteration, which is helpful for saving computational costs.

Algorithm 1 BFL-PSO algorithm

```
1: Initialize  $N$  particles, including velocities  $\mathbf{v}_i$ , positions  $\mathbf{x}_i$ , and personal best positions  $\mathbf{pbest}_i$ ;
2: Evaluate the particles  $f(\mathbf{x}_i)$ ,  $i = 1, \dots, N$ ;
3: Store the global best position  $\mathbf{gbest}$ .
4: while the terminal condition is not satisfied do
5:   %% Employed learning %%
6:   for each index  $i = 1 \rightarrow N$  do
7:     Generate the learning exemplar index  $\tau_i$  by biogeography-based exemplar generation;
8:     Update the velocity  $\mathbf{v}_i$  and the position  $\mathbf{x}_i$  using Eq.(4);
9:     Evaluate the new position  $f(\mathbf{x}_i^{new})$ ;
10:    if  $\mathbf{x}_i^{new}$  is better than  $\mathbf{pbest}_i$  then
11:       $\mathbf{pbest}_i = \mathbf{x}_i^{new}$ ,  $count(i) = 0$ 
12:    else
13:       $count(i) = count(i) + 1$ ;
14:    end if
15:  end for
16:  %% Onlooker learning %%
17:  Calculate the fitness values  $fit(\mathbf{x}_i)$  for each particle  $\mathbf{x}_i$  using Eq.(2)
18:  Calculate the probability  $p_i$  for each particle  $\mathbf{x}_i$  using Eq.(3);
19:  for each index  $i = 1 \rightarrow N$  do
20:    Select a particle  $\mathbf{x}_s$  using the roulette method based on the probability  $p_i$ ;
21:    Generate the learning exemplar index  $\tau_s$  by biogeography-based exemplar generation;
22:    Update the velocity  $\mathbf{v}_s$  and the position  $\mathbf{x}_s$  using Eq.(4);
23:    Evaluate the new position  $f(\mathbf{x}_s^{new})$ ;
24:    if  $\mathbf{x}_s^{new}$  is better than  $\mathbf{pbest}_s$  then
25:       $\mathbf{pbest}_s = \mathbf{x}_s^{new}$ ,  $count(s) = 0$ 
26:    else
27:       $count(s) = count(s) + 1$ ;
28:    end if
29:  end for
30:  %% Scout learning %%
31:  for each index  $i = 1 \rightarrow N$  do
32:    if  $count(i) \geq limit$  then
33:      Reinitialize the particle randomly, including its position  $\mathbf{x}_i$ , velocity  $\mathbf{v}_i$ , and personal best position  $\mathbf{pbest}_i$ ;
34:      Reset its counter  $count(i)=0$ ;
35:    end if
36:  end for
37: end while
```

4. Performance Evaluations

In this section, the performance of the proposed BFL-PSO algorithm is evaluated on the CEC2014 benchmark functions. These functions have complex properties such as ruggedness, noise in fitness, multimodality, ill-conditioning, interdependence and non-separability. These functions include unimodal functions (F01~F03), simple multimodal functions (F04~F16), hybrid functions (F17~F22) and composite functions (F23~F30). The detail formulations of CEC2014 benchmark functions can be found in [17].

219 The experimental studies include seven parts:

- 220 (1) Subsection 4.1 presents the parameter analysis for BFL-PSO.
- 221 (2) Subsection 4.2 analyzes the effectiveness of two new search phases introduced in BFL-PSO.
- 222 (3) Subsection 4.3 compares BFL-PSO with five state-of-the-art PSO algorithms.
- 223 (4) Subsection 4.4 compares BFL-PSO with five well-established ABC algorithms.
- 224 (5) Subsection 4.5 compares BFL-PSO with two hybrid PSO-ABC algorithms.
- 225 (6) Subsection 4.6 presents the Friedman rank test and time complexity analysis.
- 226 (7) Subsection 4.7 presents the convergence analysis.

227 The information for the compared algorithms is presented in Table 1, and the parameter
 228 settings are presented in Table 2, which are based on the corresponding literature. Moreover,
 229 each algorithm is run 51 times independently. The maximum number of function evaluations
 230 ($maxFES$) is set to be $10000 \times D$, i.e., 300000 for $D = 30$, where D is the problem dimension. The
 231 mean and standard deviations of the function error values are employed to evaluate the solution
 232 accuracy and robustness. Moreover, nonparametric statistical test methods are conducted on
 233 the experimental results to reach a reliable statistic conclusion.

Table 1: List of the compared algorithms

No.	Abbreviation	Full name	literature
1	BFL-PSO	Bee-foraging learning PSO	This work
2	CLPSO	Comprehensive learning PSO	Liang et al. [6]
3	SRPSO	Self regulating PSO	Tanweer et al. [9]
4	SLPSO	Social learning PSO	Cheng et al. [8]
5	EPSO	Ensemble PSO	Lynn et al. [10]
6	BLPSO	Biogeography-based learning PSO	Chen et al. [11]
7	ABC	Artificial bee colony	Karaboga et al. [46]
8	GABC	Gbest-guided ABC	Zhu et al. [47]
9	qABC	Quick ABC	Karaboga et al. [48]
10	ABCVSS	ABC with variable search strategy	Kiran et al. [49]
11	DFSABC	ABC with depth-first search framework and elite-guided search equations	Cui et al. [50]
12	PS-MEABC	Particle swarm inspired multi-elitist ABC	Xiang et al. [43]
13	PS-ABC	Hybrid particle swarm and ABC	Li et al. [39]

Table 2: Parameter settings for the compared algorithms

No.	Algorithm	Parameter settings
1	BFL-PSO	population size $N = 40$, inertia weight $w = 0.9 \sim 0.2$, acceleration coefficient $c = 1.49445$, maximum migration rates $I = E = 1$, refreshing gap $G = 5$, $limit = 0.5 \cdot N \cdot D$, quadratic migration model
2	CLPSO	$N = 40$, $w = 0.9 \sim 0.2$, $c = 1.49445$, refreshing gap $G = 5$
3	SRPSO	$N = 40$, $w = 1.05 \sim 0.5$, $c_1 = c_2 = 1.49445$,
4	SLPSO	$N = 100 + \text{floor}(D/10)$, social influence factor $\varepsilon = 0.0001D$
5	EPSO	$N_1 = 15, N_2 = 25$, strategy number $ns = 5$, learning period $LR = 50$
6	BLPSO	$N = 40$, $w = 0.9 \sim 0.2$, $c = 1.49445$, $I = E = 1$, refreshing gap $G = 5$, quadratic migration model
7	ABC	$N = 50$, $limit = N \times D$
8	GABC	$N = 50$, $limit = N \times D$, $C = 1.5$
9	qABC	$N = 50$, $limit = N \times D$, neighborhood radius $r = 1$
10	ABCVSS	$N = 50$, $limit = N \times D$, number of search equations $NE = 5$
11	DFSABC	$N = 50$, $limit = N \times D$, elite solution proportion $p = 0.1$, $r = 1/p$
12	PS-MEABC	$N = 50$, $limit = N \times D$
13	PS-ABC	$N = 50$, $limit_1 = 0.005 \times N \times D$, $limit_2 = 0.025 \times N \times D$, $w = 1 \sim 0$, $c_1 = c_2 = 1.4945$

4.1. Parameter tuning

In BFL-PSO, there is a parameter *limit* in the scout learning phase which needs to be carefully tuned. The candidate values for *limit* are set as $limit = K \times N \times D$, where $K = 5, 2, 1, 0.5, 0.2, 0.1$ and 0.05 , N is the population size, and D is the problem dimension.

Table 3 presents the of Friedman ranks of BFL-PSO with different *limit* values on CEC2014 functions. The mean function error values are provides in Table A1 of Appendix 2.

From Table 3, the *limit* values range from $0.1 \times N \times D$ to $0.5 \times N \times D$ provide more favorable performance. According to the final ranks, BFL-PSO with $limit = 0.5 \times N \times D$ exhibits the overall best performance on the CEC2014 functions.

Table 3: Friedman ranks of BFL-PSO with different *limit* values

<i>limit</i>	$5 \times N \times D$	$2 \times N \times D$	$N \times D$	$0.5 \times N \times D$	$0.2 \times N \times D$	$0.1 \times N \times D$	$0.05 \times N \times D$
Friedman rank	4.82	3.92	4.23	3.25	3.48	3.88	4.42
Final rank	7	4	5	1	2	3	6

4.2. Effectiveness of new search phases introduced in BFL-PSO

BFL-PSO introduces two new search phases, i.e., onlooker learning and scout learning, to traditional one-phase-based PSO. To investigate the effectiveness of both new search phases, BFL-PSO is compared with its two partial versions: (1) the first one is a partial BFL-PSO with only employed and onlooker learning phases, i.e., BFL-PSO without scout learning phase (denoted as BFL-PSO-woSC); (2) The second one is BFL-PSO with only employed and scout learning phases, i.e., BFL-PSO without onlooker learning phase (denoted as BFL-PSO-woON).

Table 4 presents the comparison results of BFL-PSO and its two partial versions.

251 From Table 4, BLF-PSO performs significantly better, similar and worse than BFL-PSO-
252 woSC on 10, 17, and 3 functions, respectively. Moreover, BFL-PSO achieves much better results
253 than BFL-PSO-woSC on the simple multimodal and composite functions. These observations
254 indicate the effectiveness of scout learning in BFL-PSO in contrast to BFL-PSO-woSC without
255 scout learning.

256 BLF-PSO performs significantly better, similar and worse than BFL-PSO-woON on 15, 13,
257 and 2 functions, respectively. Moreover, BFL-PSO achieves much better results than BFL-
258 PSO-woON on the simple multimodal and composite functions. These observations indicate the
259 effectiveness of onlooker learning in BFL-PSO in contrast to BFL-PSO-woON without onlooker
260 learning.

261 Fig.2 plots the convergence graphs of BFL-PSO and BFL-PSO-woSC without scout learning
262 on functions F08, F10, F26 and F27. The scout learning in BFL-PSO re-initializes the positions
263 and velocities of those exhausted particles, which brings new swarm diversity to prevent stagna-
264 tion. Therefore, BFL-PSO exhibits exploration search in comparison with BFL-PSO-woSC, and
265 achieves more accurate solutions on these functions.

266 Fig.3 plots the convergence graphs of BFL-PSO and BFL-PSO-woON without onlooker learn-
267 ing on functions F03, F07, F27 and F30. The onlooker learning in BFL-PSO selects those
268 particles with better fitness to perform strengthened search. Therefore, BFL-PSO exhibits ex-
269 ploitation search in comparison with BFL-PSO-woON, which improves the solution quality on
270 these functions.

271 In summary, based on these analyses, it can be said that both onlooker learning and scout
272 learning phases play important roles in the search process of BLF-PSO. The onlooker learning can
273 enhance the exploitation, while the scout learning can enhance the exploration. Both onlooker
274 and scout learning are important for the performance enhancement of BFL-PSO.

Table 4: Comparison results of BFL-PSO with its two partial versions

	BFL-PSO		BFL-PSO-woScout			BFL-PSO-woOnlooker		
	Mean	SD	Mean	SD		Mean	SD	
F01	3.26E+06	(1.48E+06)	2.94E+06	(2.04E+06)	=	3.81E+06	(1.88E+06)	=
F02	9.09E+03	(6.24E+03)	8.37E+03	(7.48E+03)	=	9.18E+03	(7.05E+03)	=
F03	1.50E+01	(3.67E+01)	4.02E+01	(1.31E+02)	=	2.31E+02	(5.65E+02)	+
F04	8.79E+01	(2.48E+01)	9.14E+01	(2.47E+01)	=	9.90E+01	(2.63E+01)	+
F05	2.03E+01	(5.25E-02)	2.03E+01	(4.64E-02)	-	2.03E+01	(5.40E-02)	-
F06	1.38E+00	(8.75E-01)	1.59E+00	(1.78E+00)	=	2.04E+00	(9.26E-01)	+
F07	0.00E+00	(0.00E+00)	1.45E-04	(1.04E-03)	=	4.35E-04	(1.76E-03)	=
F08	3.91E-01	(5.29E-01)	1.40E+00	(1.09E+00)	+	8.39E-01	(8.05E-01)	+
F09	1.69E+01	(3.12E+00)	2.41E+01	(3.99E+00)	+	1.90E+01	(4.09E+00)	+
F10	1.42E+01	(1.63E+01)	2.45E+01	(2.81E+01)	+	2.02E+01	(2.84E+01)	=
F11	1.29E+03	(2.68E+02)	1.56E+03	(2.88E+02)	+	1.42E+03	(3.59E+02)	+
F12	3.45E-01	(7.05E-02)	3.10E-01	(5.25E-02)	-	2.87E-01	(5.22E-02)	-
F13	8.57E-02	(1.41E-02)	1.60E-01	(2.60E-02)	+	8.60E-02	(1.94E-02)	=
F14	2.11E-01	(3.96E-02)	2.25E-01	(3.07E-02)	+	2.02E-01	(3.83E-02)	=
F15	3.54E+00	(4.37E-01)	4.06E+00	(5.32E-01)	+	3.48E+00	(4.39E-01)	=
F16	7.34E+00	(7.15E-01)	8.80E+00	(6.02E-01)	+	7.66E+00	(5.48E-01)	+
F17	3.59E+05	(2.31E+05)	3.15E+05	(1.70E+05)	=	4.27E+05	(2.88E+05)	=
F18	1.04E+03	(1.54E+03)	8.75E+02	(1.29E+03)	=	1.28E+03	(1.77E+03)	=
F19	4.56E+00	(9.88E-01)	4.44E+00	(1.08E+00)	=	5.02E+00	(1.20E+00)	=
F20	1.06E+03	(1.08E+03)	5.58E+02	(6.98E+02)	-	9.51E+02	(1.25E+03)	=
F21	6.21E+04	(5.14E+04)	4.73E+04	(4.05E+04)	=	6.13E+04	(5.07E+04)	=
F22	9.56E+01	(6.23E+01)	1.18E+02	(7.40E+01)	=	1.08E+02	(6.23E+01)	=
F23	3.15E+02	(9.53E-10)	3.15E+02	(8.15E-11)	=	3.15E+02	(5.10E-06)	+
F24	2.24E+02	(6.82E-01)	2.24E+02	(1.10E+00)	=	2.25E+02	(1.42E+00)	+
F25	2.05E+02	(4.75E-01)	2.05E+02	(7.04E-01)	=	2.05E+02	(6.36E-01)	+
F26	1.00E+02	(1.99E-02)	1.02E+02	(1.40E+01)	+	1.00E+02	(1.02E-01)	+
F27	3.52E+02	(4.46E+01)	3.73E+02	(4.33E+01)	+	3.74E+02	(3.99E+01)	+
F28	8.07E+02	(3.58E+01)	8.03E+02	(3.42E+01)	=	8.19E+02	(4.30E+01)	+
F29	1.29E+03	(2.78E+02)	1.21E+03	(3.35E+02)	=	1.48E+03	(3.39E+02)	+
F30	2.19E+03	(6.20E+02)	2.18E+03	(6.69E+02)	=	2.91E+03	(8.58E+02)	+
F01~F03	+/-/-		0-3-0			1-2-0		
F04~F16	+/-/-		8-3-2			6-5-2		
F17~F22	+/-/-		0-5-1			0-6-0		
F23~F30	+/-/-		2-6-0			8-0-0		
Total			10-17-3			15-13-2		

The symbols +, = and - mean that BFL-PSO is significantly superior, similar or inferior to its competitor, respectively, according to the Wilcoxon rank sum test at $\alpha = 0.05$ significance level.

The best results in terms of mean function error are highlighted in bold.

Figure 2: Convergence graphs of BFL-PSO and BFL-PSO without scout learning.

Figure 3: Convergence graphs of BFL-PSO and BFL-PSO without onlooker learning.

275 4.3. BFL-PSO vs. five state-of-the-art PSO

276 The proposed BFL-PSO is compared with five state-of-the-art PSO algorithms, i.e., CLPSO
277 [6]², SRPSO [9]³, SLPSO [8]⁴, EPSO [10], and BLPSO [11]⁵. CLPSO employs a compre-
278 hensive learning strategy where each particle learns from all particles' personal best positions
279 [6]. SRPSO uses a self-regulating inertia weight and self-perception global search direction [9].
280 SLPSO introduces a social learning mechanism where each particle learns from different better
281 particles in different dimension [8]. EPSO employs an ensemble of five different PSO updating
282 strategies [10]. BLPSO uses a biogeography-based learning strategy where each particle updates
283 itself through the biogeography-based migration [11]. It is worth noting that BLPSO is the same
284 as BFL-PSO when both onlooker learning and scout learning phases in BFL-PSO are removed.

285 Table 5 presents the comparison results of BFL-PSO and the five PSO algorithms.

286 For the unimodal functions, EPSO achieves the best results on F01 and F03. CLPSO achieves
287 the best results on F02. The performance of BFL-PSO is not satisfactory.

288 For the simple multimodal functions, BFL-PSO achieves the best results on 6 functions
289 (F07, F09 and F13~F16). CLPSO, SLPSO and BLPSO achieve the best results on 2 functions,
290 respectively. EPSO achieves the best results on 1 function.

291 For the hybrid functions, EPSO achieves the best results on 3 functions. BFL-PSO, CLPSO
292 and BLPSO achieve the best results on 1 function, respectively.

293 For the composite functions, BFL-PSO achieves the best results on 4 functions (F25~F28).
294 EPSO achieves the best results on 3 functions (F23, F29 and F30). SRPSO, SLPSO and BLPSO
295 achieve the best results on 1 function, respectively.

296 Considering all the functions, according to the Wilcoxon rank sum test in the last row, BFL-
297 PSO performs significantly better than CLPSO, SRPSO, SLPSO, EPSO and BLPSO on 21, 21,
298 18, 17 and 13 functions, respectively. By contrast, BFL-PSO performs worse than them on 6, 5,
299 6, 11 and 3 functions, respectively.

300 The above comparisons show that BFL-PSO achieves the overall best results on CEC2014
301 functions in comparison with the five PSO algorithms. Although BFL-PSO doesn't seem very
302 efficient for the unimodal functions, it achieves very good performance for the simple multimodal
303 and composite functions. For hybrid functions, the performance of BFL-PSO is worse than

²The source codes of CLPSO and EPSO can be download from <https://www.ntu.edu.sg/home/EPNSugan/>.

³The source code of SRPSO is provided by Dr. M.R. Tanweer.

⁴The source code of SLPSO can be download from <https://github.com/ranchengcn/>.

⁵The source code of BLPSO can be download from <https://github.com/chenxu796/BLPSO>.

EPHO, while better than other four PSO algorithms. These comparisons indicates BFL-PSO has potential in addressing problems with complex landscapes compared with these PSO algorithms.

4.4. BFL-PSO vs. five well-established ABC

The proposed BFL-PSO is further compared with five well-established ABC algorithms, i.e., basic ABC [46]⁶, GABC [47], qABC [48], ABCVSS [49], and DFSABC [50]. ABC uses three bee search phases and one-variable-based updating search equation to find good solutions [46]. GABC introduces the global best solution into the search equation to improve the exploitation [47]. qABC searches around the best solution among neighbors in the onlooker bee phase [48]. ABCVSS employs five different search strategies to update the positions of food sources [49]. DFSABC applies a depth-first search framework and two elite-guided search equations to ABC [50].

Table 6 presents the comparison results of BFL-PSO and the five ABC algorithms.

For the unimodal functions, BFL-PSO achieves the best result on F03. qABC and DFSABC achieve the best results on F01 and F02, respectively.

For the simple multimodal functions, BFL-PSO achieves the best results on 7 functions (F06, F07, F9, F11, F13, F15 and F16). qABC achieves the best results on 4 functions (F04, F05, F08 and F12). ABC, ABCVSS, GABC and DFSABC achieve the best results on 2, 2, 1 and 1 functions, respectively.

For the hybrid functions, BFL-PSO achieves the best results on 5 functions (F17, F19~F22). ABCVSS achieves the best results on 1 function.

For the composite functions, BFL-PSO achieves the best results on 6 functions (F23, F25~F28 and F30). qABC and ABCVSS achieve the best results on 1 function, respectively.

Considering all the functions, according to the Wilcoxon rank sum test in the last row, BFL-PSO is superior to ABC, GABC, qABC, ABCVSS, and DFSABC on 21, 22, 19, 23 and 21 functions respectively. By contrast, BFL-PSO is inferior to ABC, GABC, qABC, ABCVSS, and DFSABC on 8, 8, 7, 6 and 9 functions, respectively.

The above comparisons show that BFL-PSO performs significantly better than the five ABC algorithms on most of CE2014 functions, especially on the hybrid and composite functions.

⁶The source code of ABC can be download from <https://abc.erciyes.edu.tr/software.htm>.

Table 5: Comparison results of BFL-PSO with five state-of-the-art PSO algorithms on CEC 2014 benchmark functions

	BFL-PSO	CLPSO	SRPSO	SLPSO	EPSO	BLPSO
F01	3.26E+06 (1.48E+06)	8.91E+06 (2.90E+06)	1.60E+06 (1.60E+06)	3.86E+05 (2.60E+05)	1.53E+05 (1.17E+05)	3.56E+06 (1.68E+06)
F02	9.09E+03 (6.24E+03)	6.95E+01 (1.26E+02)	3.00E+03 (3.54E+03)	1.09E+04 (9.31E+03)	1.01E+02 (2.23E+02)	9.32E+03 (7.04E+03)
F03	1.50E+01 (3.67E+01)	1.43E+02 (1.56E+02)	5.25E+01 (5.61E+01)	6.93E+03 (4.99E+03)	6.36E-01 (9.13E-01)	9.42E+01 (2.42E+02)
F04	8.79E+01 (2.48E+01)	7.06E+01 (1.95E+01)	1.09E+02 (3.47E+01)	4.26E+01 (3.58E+01)	3.80E+00 (1.41E+01)	8.93E+01 (2.44E+01)
F05	2.03E+01 (5.25E-02)	2.03E+01 (3.95E-02)	2.09E+01 (6.50E-02)	2.09E+01 (5.02E-02)	2.05E+01 (6.15E-02)	2.03E+01 (4.12E-02)
F06	1.38E+00 (8.75E-01)	1.30E+01 (1.42E+00)	2.63E+00 (2.19E+00)	8.43E-01 (1.08E+00)	6.23E+00 (2.64E+00)	1.34E+00 (1.43E+00)
F07	0.00E+00 (0.00E+00)	6.32E-05 (1.16E-04)	1.03E-02 (1.05E-02)	5.32E-04 (2.17E-03)	3.43E-03 (5.00E-03)	1.93E-04 (1.38E-03)
F08	3.91E-01 (5.29E-01)	0.00E+00 (0.00E+00)	3.04E+01 (8.18E+00)	1.62E+01 (4.47E+00)	5.85E-02 (2.36E-01)	1.23E+00 (1.33E+00)
F09	1.69E+01 (3.12E+00)	5.42E+01 (6.67E+00)	4.02E+01 (1.22E+01)	1.93E+01 (1.32E+01)	4.42E+01 (1.21E+01)	2.49E+01 (4.75E+00)
F10	1.42E+01 (1.63E+01)	1.49E-01 (3.43E-02)	6.94E+02 (2.61E+02)	4.23E+02 (2.10E+02)	1.75E+01 (1.78E+01)	3.21E+01 (4.02E+01)
F11	1.29E+03 (2.68E+02)	2.23E+03 (2.95E+02)	2.00E+03 (6.15E+02)	8.85E+02 (3.91E+02)	1.94E+03 (4.31E+02)	1.75E+03 (2.78E+02)
F12	3.45E-01 (7.05E-02)	3.44E-01 (5.15E-02)	2.38E+00 (3.93E-01)	2.19E+00 (5.61E-01)	4.99E-01 (1.03E-01)	3.00E-01 (6.51E-02)
F13	8.57E-02 (1.41E-02)	2.94E-01 (4.14E-02)	2.52E-01 (4.52E-02)	1.72E-01 (3.65E-02)	2.75E-01 (4.06E-02)	1.60E-01 (2.68E-02)
F14	2.11E-01 (3.96E-02)	2.66E-01 (2.98E-02)	2.21E-01 (3.03E-02)	4.09E-01 (7.70E-02)	2.39E-01 (3.80E-02)	2.27E-01 (3.51E-02)
F15	3.54E+00 (4.37E-01)	8.14E+00 (9.27E-01)	8.41E+00 (4.81E+00)	6.11E+00 (4.72E+00)	5.82E+00 (3.80E+00)	4.08E+00 (6.38E-01)
F16	7.34E+00 (7.15E-01)	1.02E+01 (3.63E-01)	1.11E+01 (4.98E-01)	1.20E+01 (2.97E-01)	1.02E+01 (6.93E-01)	8.71E+00 (5.50E-01)
F17	3.59E+05 (2.31E+05)	9.24E+05 (5.07E+05)	1.90E+05 (1.18E+05)	9.19E+04 (6.03E+04)	4.15E+04 (4.35E+04)	4.92E+05 (2.71E+05)
F18	1.04E+03 (1.54E+03)	8.39E+01 (3.36E+01)	3.77E+03 (4.49E+03)	2.59E+03 (3.65E+03)	2.73E+02 (2.01E+02)	1.06E+03 (1.40E+03)
F19	4.56E+00 (9.88E-01)	7.68E+00 (6.78E-01)	6.61E+00 (1.53E+00)	8.05E+00 (8.23E+00)	6.07E+00 (1.28E+00)	4.43E+00 (8.91E-01)
F20	1.06E+03 (1.08E+03)	2.88E+03 (1.90E+03)	2.77E+02 (7.71E+01)	2.18E+04 (1.30E+04)	2.54E+02 (1.20E+02)	4.62E+02 (5.93E+02)
F21	6.21E+04 (5.14E+04)	9.42E+04 (5.62E+04)	6.42E+04 (4.40E+04)	9.48E+04 (7.65E+04)	2.85E+04 (3.86E+04)	8.52E+04 (9.09E+04)
F22	9.56E+01 (6.23E+01)	2.04E+02 (5.69E+01)	2.72E+02 (9.44E+01)	1.61E+02 (8.89E+01)	2.22E+02 (8.94E+01)	9.91E+01 (6.81E+01)
F23	3.15E+02 (9.53E-10)	3.15E+02 (6.13E-05)	3.15E+02 (6.26E-02)	3.15E+02 (1.16E-12)	3.15E+02 (1.50E-12)	3.15E+02 (2.01E-10)
F24	2.24E+02 (6.82E-01)	2.24E+02 (7.10E+00)	2.06E+02 (9.85E+00)	2.30E+02 (6.69E+00)	2.24E+02 (2.58E+00)	2.24E+02 (7.66E-01)
F25	2.05E+02 (4.75E-01)	2.08E+02 (9.83E-01)	2.05E+02 (9.79E-01)	2.05E+02 (1.39E+00)	2.06E+02 (1.68E+00)	2.05E+02 (6.37E-01)
F26	1.00E+02 (1.99E-02)	1.00E+02 (7.01E-02)	1.36E+02 (4.77E+01)	1.12E+02 (3.25E+01)	1.08E+02 (2.71E+01)	1.00E+02 (2.80E-02)
F27	3.52E+02 (4.46E+01)	4.14E+02 (5.09E+00)	4.38E+02 (7.65E+01)	3.75E+02 (6.14E+01)	4.02E+02 (8.17E-01)	3.57E+02 (4.62E+01)
F28	8.07E+02 (3.58E+01)	9.05E+02 (3.90E+01)	9.87E+02 (2.44E+02)	8.87E+02 (4.43E+01)	9.35E+02 (5.98E+01)	8.17E+02 (2.42E+01)
F29	1.29E+03 (2.78E+02)	9.89E+02 (1.34E+02)	9.17E+05 (3.70E+06)	1.89E+03 (5.77E+02)	9.21E+02 (1.14E+02)	1.42E+03 (3.53E+02)
F30	2.19E+03 (6.20E+02)	3.69E+03 (1.02E+03)	2.38E+03 (8.74E+02)	3.20E+03 (1.17E+03)	1.77E+03 (5.07E+02)	2.52E+03 (7.86E+02)
F01~F03	+/-	2-0-1	1-0-2	1-1-1	0-1-2	0-3-0
F04~F16	+/-	8-3-2	12-1-0	7-3-3	10-1-2	8-3-2
F17~F22	+/-	5-0-1	3-1-2	4-1-1	2-0-4	1-4-1
F23~F30	+/-	6-0-2	5-2-1	6-1-1	5-0-3	4-4-0
Total		21-3-6	21-4-5	18-6-6	17-2-11	13-14-3

The symbols +, = and - mean that BFL-PSO is significantly superior, similar or inferior to its competitor, respectively, according to the Wilcoxon rank sum test at $\alpha = 0.05$ significance level.
The best results in terms of mean function error are highlighted in bold.

Table 6: Comparison Results of BFL-PSO with four well-established ABC algorithms on CEC 2014 benchmark functions

	BFL-PSO	ABC	GABC	qABC	ABCVSS	DFSABC
F01	3.26E+06 (1.48E+06)	5.83E+06 (2.45E+06)	5.10E+06 (2.26E+06)	3.09E+06 (1.71E+06)	4.75E+06 (2.17E+06)	4.29E+06 (2.30E+06)
F02	9.09E+03 (6.24E+03)	1.02E+02 (1.68E+02)	1.10E+02 (1.18E+02)	1.21E+02 (1.31E+02)	9.15E+01 (1.26E+02)	8.88E+02 (1.54E+03)
F03	1.50E+01 (3.67E+01)	7.94E+02 (6.65E+02)	5.91E+02 (5.38E+02)	6.78E+02 (7.77E+02)	5.16E+02 (5.10E+02)	8.47E+02 (8.51E+02)
F04	8.79E+01 (2.48E+01)	2.47E+01 (2.52E+01)	3.37E+01 (2.89E+01)	1.32E+01 (2.34E+01)	2.01E+01 (2.40E+01)	3.58E+01 (3.85E+01)
F05	2.03E+01 (5.25E-02)	2.01E+01 (1.93E-02)	2.01E+01 (3.37E-02)	2.00E+01 (1.13E-02)	2.01E+01 (2.88E-02)	2.01E+01 (3.05E-02)
F06	1.38E+00 (8.75E-01)	1.46E+01 (1.60E+00)	1.31E+01 (1.26E+00)	1.36E+01 (1.63E+00)	1.36E+01 (1.39E+00)	1.16E+01 (1.42E+00)
F07	0.00E+00 (0.00E+00)	1.66E-05 (3.01E-05)	7.12E-06 (9.21E-06)	2.81E-07 (3.35E-07)	2.06E-05 (1.91E-05)	1.53E-03 (1.09E-02)
F08	3.91E-01 (5.29E-01)	0.00E+00 (0.00E+00)	0.00E+00 (0.00E+00)	0.00E+00 (0.00E+00)	0.00E+00 (0.00E+00)	0.00E+00 (0.00E+00)
F09	1.69E+01 (3.12E+00)	8.24E+01 (1.24E+01)	5.65E+01 (8.87E+00)	8.30E+01 (1.41E+01)	5.34E+01 (8.80E+00)	4.17E+01 (7.34E+00)
F10	1.42E+01 (1.63E+01)	9.97E-01 (6.49E-01)	4.18E-01 (3.89E-01)	6.90E-02 (3.57E-02)	4.72E-02 (2.96E-02)	7.55E-01 (9.51E-01)
F11	1.29E+03 (2.68E+02)	1.92E+03 (2.00E+02)	1.72E+03 (2.48E+02)	1.84E+03 (2.58E+02)	1.70E+03 (2.37E+02)	1.56E+03 (2.55E+02)
F12	3.45E-01 (7.05E-02)	1.63E-01 (2.15E-02)	1.88E-01 (3.60E-02)	1.12E-01 (2.46E-02)	1.76E-01 (3.23E-02)	1.40E-01 (2.57E-02)
F13	8.57E-02 (1.41E-02)	2.26E-01 (3.39E-02)	2.36E-01 (3.14E-02)	2.34E-01 (2.92E-02)	2.25E-01 (3.66E-02)	1.90E-01 (3.27E-02)
F14	2.11E-01 (3.96E-02)	1.96E-01 (1.54E-02)	1.97E-01 (2.04E-02)	2.01E-01 (1.50E-02)	2.22E-01 (1.96E-02)	2.30E-01 (2.32E-02)
F15	3.54E+00 (4.37E-01)	8.35E+00 (1.38E+00)	6.19E+00 (1.11E+00)	6.86E+00 (1.24E+00)	6.35E+00 (1.03E+00)	4.14E+00 (7.30E-01)
F16	7.34E+00 (7.15E-01)	9.77E+00 (4.02E-01)	9.34E+00 (3.96E-01)	9.26E+00 (5.16E-01)	9.45E+00 (3.50E-01)	8.63E+00 (5.28E-01)
F17	3.59E+05 (2.31E+05)	1.87E+06 (8.60E+05)	1.90E+06 (9.35E+05)	9.95E+05 (5.58E+05)	1.71E+06 (1.04E+06)	1.58E+06 (8.57E+05)
F18	1.04E+03 (1.54E+03)	6.93E+02 (5.92E+02)	3.23E+03 (2.85E+03)	3.27E+02 (1.88E+02)	2.83E+02 (3.89E+02)	1.58E+03 (2.14E+03)
F19	4.56E+00 (9.88E-01)	7.23E+00 (6.99E-01)	6.77E+00 (6.79E-01)	6.99E+00 (7.95E-01)	6.90E+00 (7.33E-01)	6.31E+00 (8.97E-01)
F20	1.06E+03 (1.08E+03)	8.37E+03 (4.33E+03)	7.95E+03 (4.15E+03)	7.27E+03 (4.26E+03)	7.90E+03 (3.45E+03)	7.18E+03 (3.75E+03)
F21	6.21E+04 (5.14E+04)	1.70E+05 (9.99E+04)	2.24E+05 (1.62E+05)	1.54E+05 (1.16E+05)	1.35E+05 (1.01E+05)	2.09E+05 (1.39E+05)
F22	9.56E+01 (6.23E+01)	3.00E+02 (9.85E+01)	2.58E+02 (9.56E+01)	3.03E+02 (1.04E+02)	2.54E+02 (9.75E+01)	2.16E+02 (1.04E+02)
F23	3.15E+02 (9.53E-10)	3.15E+02 (5.97E-02)	3.15E+02 (1.06E-01)	3.15E+02 (4.47E-02)	3.15E+02 (1.09E-01)	3.15E+02 (1.02E-01)
F24	2.24E+02 (6.82E-01)	2.26E+02 (5.03E+00)	2.24E+02 (1.03E+01)	2.26E+02 (6.80E+00)	2.23E+02 (1.14E+01)	2.26E+02 (9.01E-01)
F25	2.05E+02 (4.75E-01)	2.09E+02 (1.18E+00)	2.08E+02 (1.16E+00)	2.06E+02 (1.28E+00)	2.08E+02 (8.93E-01)	2.07E+02 (1.08E+00)
F26	1.00E+02 (1.99E-02)	1.00E+02 (5.80E-02)	1.00E+02 (4.65E-02)	1.00E+02 (7.10E-02)	1.00E+02 (5.62E-02)	1.00E+02 (4.25E-02)
F27	3.52E+02 (4.46E+01)	4.02E+02 (5.62E+01)	4.02E+02 (5.62E+01)	4.07E+02 (2.90E+00)	4.11E+02 (3.74E+00)	4.08E+02 (4.07E+00)
F28	8.07E+02 (3.58E+01)	1.22E+03 (1.60E+02)	9.35E+02 (5.86E+01)	1.19E+03 (1.72E+02)	9.43E+02 (5.07E+01)	8.79E+02 (4.80E+01)
F29	1.29E+03 (2.78E+02)	1.01E+03 (7.28E+01)	1.12E+03 (1.57E+02)	9.64E+02 (6.98E+01)	1.14E+03 (1.06E+02)	1.25E+03 (3.19E+02)
F30	2.19E+03 (6.20E+02)	2.69E+03 (6.43E+02)	2.77E+03 (7.67E+02)	2.26E+03 (5.97E+02)	3.20E+03 (8.76E+02)	2.94E+03 (7.87E+02)
F01~F03	+/-/-	2-0-1	2-0-1	1-1-1	2-0-1	2-0-1
F04~F16	+/-/-	7-0-6	7-0-6	7-1-5	8-0-5	8-0-5
F17~F22	+/-/-	5-1-0	6-0-0	5-1-0	6-0-0	5-0-1
F23~F30	+/-/-	7-0-1	7-0-1	6-1-1	7-1-0	6-0-2
Total		21-1-8	22-0-8	19-4-7	23-1-6	21-0-9

The symbols +, = and - mean that BFL-PSO is significantly superior, similar or inferior to its competitor, respectively, according to the Wilcoxon rank sum test at $\alpha = 0.05$ significance level.

The best results in terms of mean function error are highlighted in bold.

332 4.5. BFL-PSO vs. two hybrid PSO-ABC algorithms

333 The BFL-PSO is further compared with two recently-developed hybrid PSO-ABC algorithms,
334 i.e., PS-MEABC [43] and PS-ABC [39]. PS-MEABC employs the global best solution and an
335 elitist archive strategy to modify the solutions in ABC [43]. PS-ABC combines one local search
336 phase from PSO and two search phases from ABC to form a hybrid algorithm [39].

337 Table 7 presents the comparison results of BFL-PSO and the hybrid PSO-ABC algorithms.

338 First, BFL-PSO is compared with PS-MEABC. For the unimodal functions, BFL-PSO per-
339 forms better than PS-MEABC on none function, but worse on 2 functions. For the simple
340 multimodal, BFL-PSO performs better than PS-MEABC on 7 functions, but worse on 6 func-
341 tions. For the hybrid functions BFL-PSO performs better than PS-MEABC on 2 functions, but
342 worse on 2 functions. For the composite functions, BFL-PSO performs better than PS-MEABC
343 on 6 functions, but worse on 1 function.

344 Then, BFL-PSO is compared with PS-ABC. For the unimodal functions, BFL-PSO performs
345 better than PS-ABC on 1 function, but worse on 1 function. For the simple multimodal, BFL-
346 PSO performs better than PS-ABC on 13 functions, but worse on none function. For the hybrid
347 functions BFL-PSO performs better than PS-ABC on 5 functions, but worse on none function.
348 For the composite functions, BFL-PSO performs better than PS-ABC on 8 functions, but worse
349 on none function.

350 In summary, the above comparisons show that BFL-PSO exhibits competitive performance
351 compared with two recently hybrid PSO-ABC algorithms. In particular, BFL-PSO achieves
352 much better results on the complex composite functions.

Table 7: Comparison results of BFL-PSO with two hybrid PSO-ABC algorithms on CEC 2014 benchmark functions

	BFL-PSO		PS-MEABC			PS-ABC		
	Mean	SD	Mean	SD		Mean	SD	
F01	3.26E+06	(1.48E+06)	3.21E+06	(1.43E+06)	=	5.62E+06	(6.75E+06)	=
F02	9.09E+03	(6.24E+03)	0.00E+00	(0.00E+00)	-	3.30E+02	(3.95E+02)	-
F03	1.50E+01	(3.67E+01)	0.00E+00	(0.00E+00)	-	5.52E+02	(1.02E+03)	+
F04	8.79E+01	(2.48E+01)	6.84E+01	(3.34E+01)	-	1.41E+02	(7.46E+01)	+
F05	2.03E+01	(5.25E-02)	2.02E+01	(3.78E-02)	-	2.09E+01	(4.48E-02)	+
F06	1.38E+00	(8.75E-01)	1.31E+01	(1.52E+00)	+	1.81E+01	(3.30E+00)	+
F07	0.00E+00	(0.00E+00)	1.16E-05	(2.46E-05)	+	9.21E-03	(1.18E-02)	+
F08	3.91E-01	(5.29E-01)	0.00E+00	(0.00E+00)	-	5.75E+01	(1.56E+01)	+
F09	1.69E+01	(3.12E+00)	5.87E+01	(9.54E+00)	+	8.45E+01	(2.32E+01)	+
F10	1.42E+01	(1.63E+01)	2.71E+00	(1.15E+00)	-	1.85E+03	(6.15E+02)	+
F11	1.29E+03	(2.68E+02)	1.82E+03	(2.31E+02)	+	3.38E+03	(5.76E+02)	+
F12	3.45E-01	(7.05E-02)	2.65E-01	(5.24E-02)	-	1.45E+00	(6.45E-01)	+
F13	8.57E-02	(1.41E-02)	2.78E-01	(4.22E-02)	+	3.16E-01	(6.44E-02)	+
F14	2.11E-01	(3.96E-02)	1.93E-01	(2.41E-02)	-	3.98E-01	(1.54E-01)	+
F15	3.54E+00	(4.37E-01)	6.87E+00	(1.85E+00)	+	1.32E+01	(4.67E+00)	+
F16	7.34E+00	(7.15E-01)	9.83E+00	(3.08E-01)	+	1.13E+01	(6.58E-01)	+
F17	3.59E+05	(2.31E+05)	3.24E+05	(1.97E+05)	=	4.83E+05	(7.74E+05)	+
F18	1.04E+03	(1.54E+03)	1.39E+03	(1.73E+03)	=	4.29E+03	(3.78E+03)	+
F19	4.56E+00	(9.88E-01)	6.53E+00	(6.60E-01)	+	1.33E+01	(9.24E+00)	+
F20	1.06E+03	(1.08E+03)	7.82E+01	(4.82E+01)	-	4.08E+03	(3.34E+03)	+
F21	6.21E+04	(5.14E+04)	3.35E+04	(3.19E+04)	-	1.26E+05	(3.61E+05)	=
F22	9.56E+01	(6.23E+01)	1.79E+02	(8.92E+01)	+	3.80E+02	(2.06E+02)	+
F23	3.15E+02	(9.53E-10)	3.15E+02	(4.53E-06)	+	3.16E+02	(4.59E-01)	+
F24	2.24E+02	(6.82E-01)	2.27E+02	(9.00E-01)	+	2.38E+02	(6.79E+00)	+
F25	2.05E+02	(4.75E-01)	2.09E+02	(1.37E+00)	+	2.16E+02	(5.45E+00)	+
F26	1.00E+02	(1.99E-02)	1.00E+02	(4.33E-02)	+	1.31E+02	(4.56E+01)	+
F27	3.52E+02	(4.46E+01)	4.12E+02	(4.90E+00)	+	7.40E+02	(2.32E+02)	+
F28	8.07E+02	(3.58E+01)	9.33E+02	(3.74E+01)	+	1.71E+03	(5.52E+02)	+
F29	1.29E+03	(2.78E+02)	1.23E+03	(2.52E+02)	=	3.29E+06	(7.79E+06)	+
F30	2.19E+03	(6.20E+02)	1.90E+03	(5.48E+02)	-	7.88E+03	(6.67E+03)	+
F01~F03	+/-/-		0-1-2			1-1-1		
F04~F16	+/-/-		7-0-6			13-0-0		
F17~F22	+/-/-		2-2-2			5-1-0		
F23~F30	+/-/-		6-1-1			8-0-0		
Total	+/-/-		15-4-11			27-2-1		

The symbols +, = and - mean that BFL-PSO is significantly superior, similar or inferior to its competitor, respectively, according to the Wilcoxon rank sum test at $\alpha = 0.05$ significance level.
The best results in terms of mean function error are highlighted in bold.

4.6. Friedman ranks and runtime analysis

Table 8 presents the Friedman ranks of all the compared algorithms on four groups of CEC2014 functions. For the unimodal functions, EPSO and PS-MEABC achieve the best rank (2.33), and BLF-PSO achieves the fifth rank (6.67). For the simple multimodal functions, BLF-PSO achieves the best rank (4.62). For the hybrid functions, EPSO achieves the best rank (2.67), and BLF-PSO achieves the second best rank (3.67). For the composite functions, BFL-PSO achieves the best rank (2.81).

When considering all 30 functions, BFL-PSO achieves the best rank (4.15), BLPSO the second (5.38), EPSO the third (5.45), followed by PS-MEABC and others. In addition, the statistical test is conducted at $\alpha = 0.05$ significance level in Table 9. Based on the unadjusted p -value given in Table 9, BFL-PSO is significantly superior to most of the compared algorithms excluding BLPSO, EPSO and PS-MEABC.

Table 10 presents the average runtime of all the algorithms on 30 functions. Fig.4 plots the bar chart of the average runtime on function F01. BFL-PSO introduces two additional search phases (i.e., onlooker learning and scout learning), which may bring more runtime in BFL-PSO. From Fig.4, the runtime of BFL-PSO is 13.04 seconds, which is larger than SRPSO, SLPSO and DFSABC. However, the runtime of BFL-PSO is less than all the other algorithms including CLPSO, EPSO and BLPSO.

Table 8: Friedman test of mean error values on four groups of functions

Algorithm	Unimodal: F01~F03		Multimodal: F04~F16		Hybrid: F17~F22		Composition: F23~F30		Total: F01~F30	
	Rank		Rank		Rank		Rank		Rank	
BFL-PSO	6.67	5	4.62	1	3.67	2	2.81	1	4.15	1
CLPSO	7.00	6	8.31	11	6.17	5	6.88	5	7.37	9
SRPSO	5.67	3	10.15	12	6.33	6	8.50	12	8.50	12
SLPSO	9.33	11	7.69	9	7.83	7	7.19	8	7.75	10
EPSO	2.33	1	8.00	10	2.67	1	4.56	3	5.45	3
BLPSO	8.00	8	6.08	6	4.33	4	4.06	2	5.38	2
ABC	9.33	12	6.38	7	10.17	12	7.88	10	7.83	11
GABC	8.33	9	5.38	5	10.67	13	7.38	9	7.27	8
qABC	7.00	7	5.00	3	8.83	10	6.75	4	6.43	5
ABCVSS	9.67	13	4.69	2	8.17	8	7.13	6	6.53	6
DFSABC	6.33	4	5.31	4	8.17	9	7.88	11	6.67	7
PS-MEABC	2.33	2	6.92	8	4.00	3	7.13	7	5.93	4
PS-ABC	9.00	10	12.46	13	10.00	11	12.88	13	11.73	13

Table 9: Adjusted p -values obtained by Bonferroni-Dunn and Holm procedures for BFL-PSO and the compared algorithms

BFL-PSO vs.	unadjusted p	Bonferroni p	Holm p
PS-ABC	0.0000	0.0000	0.0000
SRPSO	0.0000	0.0002	0.0002
ABC	0.0002	0.0030	0.0025
SLPSO	0.0003	0.0041	0.0031
CLPSO	0.0014	0.0166	0.0110
GABC	0.0019	0.0233	0.0136
DFSABC	0.0123	0.1479	0.0739
ABCVSS	0.0178	0.2133	0.0889
qABC	0.0232	0.2779	0.0926
PS-MEABC	0.0761	0.9137	0.2284
EPSO	0.1961	2.3528	0.3921
BLPSO	0.2200	2.6400	0.3921

Table 10: The average run time (in seconds) of the compared algorithms on all CEC2014 functions

	BFL-PSO	CLPSO	SRPSO	SLPSO	EPSO	BLPSO	ABC	GABC	qABC	ABCVSS	DFSABC	PS-MEABC	PS-ABC
F01	13.04	16.40	4.88	6.39	25.01	17.42	14.58	14.87	20.71	16.53	12.57	13.99	16.59
F02	11.41	14.42	4.46	6.15	23.83	15.39	14.60	14.54	20.40	16.38	11.90	12.95	15.10
F03	11.86	14.95	4.47	5.66	24.19	16.57	14.63	14.85	19.76	16.37	11.98	12.77	15.21
F04	11.67	15.42	4.44	5.83	25.31	15.78	13.75	13.68	18.57	16.04	11.91	12.89	15.57
F05	13.27	16.49	4.65	5.95	29.02	19.73	13.73	13.82	18.47	16.56	12.23	12.99	13.73
F06	29.72	35.27	22.06	25.29	43.90	35.30	32.56	32.69	37.95	35.82	30.70	32.04	32.37
F07	11.32	14.67	4.71	5.86	24.86	16.22	13.54	13.63	18.56	16.51	11.95	12.96	13.72
F08	11.55	14.67	4.43	5.62	24.06	16.86	13.17	13.22	18.14	16.20	11.55	12.65	13.47
F09	12.39	15.85	4.65	5.76	25.53	17.67	13.51	13.63	18.50	16.28	12.00	12.93	13.83
F10	13.18	16.37	5.10	6.86	28.08	18.30	14.02	13.97	19.11	16.99	12.38	13.64	14.78
F11	13.63	17.38	5.33	6.63	28.83	20.65	14.46	14.51	19.57	17.15	12.98	14.06	15.26
F12	16.38	20.27	7.85	9.98	33.38	23.43	16.87	16.89	22.01	19.87	15.44	16.47	17.32
F13	11.69	15.45	4.44	5.49	27.58	17.20	13.22	13.29	18.28	16.28	11.83	12.68	13.20
F14	11.66	15.26	4.47	5.77	27.07	16.95	13.35	13.28	18.26	16.36	11.82	12.76	13.29
F15	11.65	15.25	4.63	5.66	24.73	16.79	13.56	13.66	18.85	16.43	12.04	12.95	14.81
F16	12.52	16.63	4.70	5.76	28.18	18.96	13.55	13.57	18.66	16.40	12.41	13.18	14.03
F17	12.86	16.62	4.94	6.07	26.74	19.02	14.80	15.03	20.70	16.91	12.84	14.17	15.98
F18	11.98	15.77	4.60	5.87	89.56	17.38	14.49	14.57	20.62	16.48	12.45	13.07	15.59
F19	15.99	20.52	8.38	10.39	118.00	21.39	17.67	17.83	22.85	20.92	16.39	17.24	18.34
F20	12.20	16.38	4.66	5.97	101.21	17.95	14.08	14.20	19.31	16.51	12.49	13.26	15.84
F21	12.80	16.48	4.91	6.29	104.67	18.89	14.87	14.91	20.69	16.77	12.77	14.01	15.47
F22	13.37	17.29	5.20	6.50	110.45	19.37	14.38	14.52	19.65	17.36	13.24	13.95	15.31
F23	16.41	19.46	8.43	10.26	129.88	20.60	17.61	17.86	22.65	20.64	16.24	17.29	20.51
F24	14.37	18.12	7.24	8.73	107.19	19.47	16.36	16.48	21.43	19.61	15.20	16.00	17.14
F25	16.12	20.60	7.86	9.48	138.22	22.32	17.15	17.38	22.22	20.17	15.97	16.72	18.55
F26	36.19	40.11	27.85	32.83	201.52	42.61	37.92	38.15	43.64	41.76	36.66	38.45	38.55
F27	34.93	40.55	27.40	29.41	193.51	41.36	37.84	38.29	43.71	41.50	36.63	37.96	38.51
F28	18.06	23.00	9.65	11.67	169.50	24.07	19.20	19.37	24.70	22.17	18.11	18.79	21.17
F29	20.67	25.03	11.10	13.02	190.03	27.31	21.53	21.66	27.40	24.52	19.77	20.68	22.87
F30	16.72	22.01	7.75	9.42	48.39	21.64	17.90	17.95	23.83	20.66	16.23	16.99	20.62

Figure 4: The average runtime (in seconds) of all the algorithms on function F01

4.7. Convergence analysis

Fig.5 plots the convergence graphs of BFL-PSO and five PSO algorithms on 12 representative functions. It can be observed that BFL-PSO has the second fastest convergence speed (only after SLPSO), but it can achieve the best solution accuracy on most functions including F07, F09, F13, F16, F22, F25, F27 and F28. SLPSO has the fastest convergence speed on most functions,

376 but its solution accuracy is worse than BFL-PSO. EPSO has moderate convergence speed, and
377 achieves the most accurate solutions on F01, F03 and F17. CLPSO and SRPSO have the slowest
378 convergence speed, and their solution accuracy is also not efficient on most functions.

379 Fig.6 plots the convergence graphs of BFL-PSO and five ABC algorithms on 12 representative
380 functions. It can be observed that BFL-PSO can convergence to the best solution accuracy
381 compared with the five ABC algorithms on most functions including F01, F03, F06, F07, F09,
382 F13, F17, F19, F21, F27 and F28.

383 Fig.7 plots the convergence graphs of BFL-PSO and two hybrid PSO-ABC algorithms on
384 12 representative functions. It can be observed that BFL-PSO has advantages on functions
385 including F06, F07, F09, F11, F13, F15, F18, F19, F22 and F27. PS-MEABC has advantages
386 on two unimodal functions F02 and F03.

387 Based on the above comparisons, it can be concluded that BFL-PSO also has competitive
388 performance in terms of convergence, and it can converge to the most accurate solutions on the
389 selected representative functions.

Figure 5: Convergence graphs of BLF-PSO and the compared PSO algorithms

Figure 6: Convergence graphs of BLF-PSO and the compared ABC algorithms

Figure 7: Convergence graphs of BLF-PSO and the hybrid PSO-ABC algorithms

5. Application to power economic dispatch problems

To verify the performance of BFL-PSO in solving real-world problem, BFL-PSO is applied to solve power economic dispatch (PED) problems.

5.1. Formulation of PED problem

The PED problem aims to minimize the fuel costs by allocating optimal power output among different generators. At the same time, various generator constraints need to be satisfied [51].

The objective function of the PED problem is formulated as:

$$\begin{aligned} \text{minimize } F &= \sum_{i=1}^{NT} F_i(P_i) \\ F_i(P_i) &= a_i + b_i P_i + c_i P_i^2 + |e_i \sin(f_i(P_i^{\min} - P_i))| \end{aligned} \quad (5)$$

where F is the total fuel cost, F_i is the fuel cost of the i -th generator, P_i is the power output of the i -th generator, and a_i , b_i , c_i , e_i and f_i are the fuel cost efficient of the i -th generator.

Power balance constraint: the total real power output of all generators should be equal to the total system demand (P_D) plus the total transmission network loss (P_L), i.e.,

$$\sum_{i=1}^{NT} P_i = P_D + P_L \quad (6)$$

Generator capacity constraint: the real power output of each generator should be within its minimum boundary ($P_i^{\min}$) and maximum boundary ($P_i^{\max}$), i.e.,

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad (7)$$

Ramp-rate constraints: the operating range of each generator is restricted by its ramp-rate limit. The increase and decrease in generation are limited by:

$$P_i - P_i^0 \leq UR_i \quad (8)$$

$$P_i^0 - P_i \leq DR_i \quad (9)$$

where P_i^0 is the previous power output, UR_i and DR_i are the up-ramp and down-ramp limits, respectively.

Prohibited operating zones (POZs): The feasible operating zones of the i -th generator are described as follows:

$$P_i \in \begin{cases} P_i^{\min} \leq P_i \leq P_{i,1}^l \\ P_{i,k-1}^u \leq P_i \leq P_{i,k}^l, \quad k = 2, 3, \dots, n_i, i = 1, 2, \dots, NT \\ P_{i,n_i}^u \leq P_i \leq P_i^{\max} \end{cases} \quad (10)$$

where n_i is the number of POZs of i -th generator. $P_{i,k}^l$ and $P_{i,k}^u$ are the lower and upper power output of the k -th POZ of the i -th generator, respectively.

5.2. Simulation results

Two PED problems are solved by the BFL-PSO algorithm. They are:

- Case 1: 6 unit system [52, 53]
- Case 2: 20 unit system [11, 54]

Table 11 presents the statistical results of BFL-PSO for the 6 unit PED problem. The results of BLPSO [52], ST-IRDPSO (improved random drift PSO) [53], MABC (modified ABC) [55] and CBA (chaotic bat algorithm) [54] reported in the literature are also given in Table 11. It can be observed that BFL-PSO achieves the best results in terms of the mean (15447.40\$/h), minimal (15447.32 \$/h) and maximal (15447.59 \$/h) fuel costs among the five algorithms. MABC gets the best standard deviation (6.04E-08), but its solution accuracy is worse than BFL-PSO.

Table 12 presents the statistical results of BFL-PSO for the 20 unit PED problem. The results of BLPSO [52], CBA [54], BSA (backtracking search algorithm) [56] and CQGSO (continuous quick group search optimizer) [57] reported in the literature are also given in Table 12. From Table 12, BFL-PSO, BLPSO and CQGSO achieve very close results in terms of mean, minimal and maximal fuel costs, i.e. about 62456 \$/h. Moreover, the standard deviation of BFL-PSO is very small.

Based on the above results, it can be said that BLF-PSO can achieve similar or even better results for the PED problems in comparison with the methods in the literature. Therefore, BFL-PSO can be used as an effective method for real-world PED problem.

Table 11: Statistical results of the fuel costs for 6 unit system

Algorithm	Mean	Min	Max	SD
BFL-PSO	15447.40	15447.32	15447.59	0.06
BLPSO [52]	15447.45	15447.34	15447.67	0.09
ST-IRDPSO [53]	15450.7	15449.89	NA	1.416
MABC [55]	15449.8995	15449.8995	15449.8995	6.04E-08
CBA [54]	15454.76	15450.2381	15518.6588	2.965

NA means not available in the literature.
The best results are highlighted in bold.

Table 12: Statistical results of the fuel costs for 20 unit system

Algorithm	Mean	Min	Max	SD
BFL-PSO	62456.64	62456.57	62456.66	0.01
BLPSO [52]	62456.64	62456.58	62456.65	0.01
CBA [54]	62456.6348	62456.6328	62501.6714	0.3879
BSA [56]	62457.1517	62456.6925	62458.1272	NA
CQGSO [57]	62456.6331	62456.633	62456.63337	NA

NA means not available in the literature.
The best results are highlighted in bold.

6. Conclusion

In this paper, a new bee-foraging learning PSO (BFL-PSO) algorithm has been proposed, which is based on the bee-foraging search mechanism. The proposed BFL-PSO has three different search phases, namely employed learning, onlooker learning and scout learning. The employed learning works like the traditional one-phased-based PSO, while the onlooker learning performs strengthened search around the high-quality particle to exploit promising solutions, and the scout learning re-initializes the exhausted particles to introduce new diversity. Therefore, all three search phases contribute to the improvement of the search performance of BFL-PSO.

The proposed BFL-PSO has been comprehensively evaluated on the CEC2014 functions, and compared with well-established swarm optimization algorithms, including five PSO algorithms, five ABC algorithms and two hybrid PSO-ABC algorithms. Experimental results demonstrated that BFL-PSO can achieve competitive or even better performance on most test functions, especially on the composite functions with complex landscape. In addition, the effectiveness of the onlooker learning and scout learning in BFL-PSO has been verified through numerical simulations.

In the future, there is much work worth of further investigation. For example, the performance of BFL-PSO is not satisfactory for unimodal functions, and thus one possible work is to combine BFL-PSO with local search technique to further enhance the performance of BFL-PSO. In addition, BFL-PSO may be extended for other types of optimization problems such as multi-objective, large-scale and real-world problems.

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454 **Appendix 1. Learning exemplar generation in BLPSO [11]**

455 Firstly, for a minimization problem, all particles in the population are sorted based on the
 456 quality of their ***pbest*** from the best to the worst :

$$f(\mathbf{pbest}_{s_1}) \leq \dots f(\mathbf{pbest}_{s_k}) \leq \dots \leq f(\mathbf{pbest}_{s_N}) \quad (11)$$

457 where s_k is the subscript of the particle with the k -th best ***pbest***; N is the population size.

458 Secondly, the ranking values of all particles are assigned as follows:

$$rank(\mathbf{x}_{s_k}) = N - k, \quad k = 1, 2, \dots, N \quad (12)$$

459 Thirdly, the immigration and emigration rates of particles are assigned as follows:

$$\begin{cases} \lambda(\mathbf{x}_{s_k}) = \left(1 - \frac{N-k}{N}\right)^2 \\ \mu(\mathbf{x}_{s_k}) = \left(\frac{N-k}{N}\right)^2 \end{cases}, \quad k = 1, 2, \dots, N \quad (13)$$

460 Finally, the learning exemplar indices $\tau_i = [\tau_i(1), \dots, \tau_i(d), \dots, \tau_i(D)]$ for the i -th particle
 461 can be generated by Algorithm 2.

Algorithm 2 Biogeography-based exemplar generation method

Input: particle index i , immigration rates λ_k , and emigration rates μ_k , $k = 1, 2, \dots, N$

Output: learning exemplar indices $\tau_i = [\tau_i(1), \dots, \tau_i(d), \dots, \tau_i(D)]$

```

1: for  $k = 1 \rightarrow D$  do
2:   if  $rand < \lambda_k$  then
3:     Utilize a roulette wheel to select a particle index  $j$  with probability  $\propto \mu_j$ ;
4:      $\tau_i(k) = j$ ;
5:   else
6:      $\tau_i(k) = i$ ;
7:   end if
8: end for
```

Table A1: Comparison results of BFL-PSO with different *limit* values

<i>limit</i>	$5 \times N \times D$	$2 \times N \times D$	$N \times D$	$0.5 \times N \times D$	$0.2 \times N \times D$	$0.1 \times N \times D$	$0.05 \times N \times D$
F01	3.52E+06	2.93E+06	2.92E+06	3.26E+06	2.13E+06	1.71E+06	2.20E+06
F02	9.50E+03	8.24E+03	1.15E+04	9.09E+03	9.84E+03	1.04E+04	9.08E+03
F03	3.55E+01	3.11E+01	1.62E+01	1.50E+01	1.90E+02	6.58E+02	9.48E+02
F04	8.43E+01	8.33E+01	8.08E+01	8.79E+01	8.91E+01	8.62E+01	9.43E+01
F05	2.03E+01	2.03E+01	2.03E+01	2.03E+01	2.04E+01	2.05E+01	2.06E+01
F06	1.27E+00	1.20E+00	1.17E+00	1.38E+00	1.19E+00	9.79E-01	7.41E-01
F07	0.00E+00	0.00E+00	3.38E-04	0.00E+00	1.45E-04	0.00E+00	1.45E-04
F08	1.44E+00	1.05E+00	8.12E-01	3.91E-01	2.37E-01	4.25E-01	5.47E-01
F09	2.36E+01	2.35E+01	2.17E+01	1.69E+01	9.53E+00	7.87E+00	1.07E+01
F10	3.52E+01	2.18E+01	1.93E+01	1.42E+01	7.29E+00	9.88E+00	1.23E+01
F11	1.68E+03	1.53E+03	1.50E+03	1.29E+03	4.98E+02	2.29E+02	2.98E+02
F12	3.17E-01	3.04E-01	3.03E-01	3.45E-01	3.98E-01	5.29E-01	7.70E-01
F13	1.53E-01	1.39E-01	1.12E-01	8.57E-02	6.95E-02	7.97E-02	1.42E-01
F14	2.38E-01	2.20E-01	2.17E-01	2.11E-01	2.45E-01	2.52E-01	2.35E-01
F15	4.31E+00	4.00E+00	3.83E+00	3.54E+00	3.35E+00	3.56E+00	8.02E+00
F16	8.58E+00	8.65E+00	8.16E+00	7.34E+00	6.00E+00	5.74E+00	7.07E+00
F17	3.37E+05	3.15E+05	3.79E+05	3.59E+05	3.43E+05	3.69E+05	4.37E+05
F18	9.53E+02	6.33E+02	1.12E+03	1.04E+03	8.72E+02	8.93E+02	7.71E+02
F19	4.39E+00	4.56E+00	4.39E+00	4.56E+00	4.42E+00	4.08E+00	3.92E+00
F20	6.82E+02	9.28E+02	6.36E+02	1.06E+03	2.18E+03	1.94E+03	2.76E+03
F21	6.41E+04	5.92E+04	6.62E+04	6.21E+04	9.83E+04	1.27E+05	1.58E+05
F22	1.08E+02	9.77E+01	9.72E+01	9.56E+01	9.05E+01	1.05E+02	1.12E+02
F23	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02
F24	2.24E+02	2.24E+02	2.24E+02	2.24E+02	2.24E+02	2.24E+02	2.22E+02
F25	2.05E+02	2.05E+02	2.05E+02	2.05E+02	2.05E+02	2.05E+02	2.05E+02
F26	1.00E+02	1.00E+02	1.02E+02	1.00E+02	1.00E+02	1.00E+02	1.00E+02
F27	3.73E+02	3.75E+02	3.69E+02	3.52E+02	3.53E+02	3.59E+02	3.29E+02
F28	8.09E+02	8.13E+02	8.14E+02	8.07E+02	8.06E+02	8.08E+02	8.08E+02
F29	1.25E+03	1.26E+03	1.34E+03	1.29E+03	1.39E+03	1.58E+03	1.47E+03
F30	2.30E+03	2.40E+03	2.27E+03	2.19E+03	2.72E+03	3.00E+03	3.17E+03
Friedman rank	4.82	3.92	4.23	3.25	3.48	3.88	4.42
Final rank	7	4	5	1	2	3	6

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