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Reliability Analysis of a Subsystem in Aluminium Plant

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Abstract – The paper deals with a specific subsystem analysis called butt shot blast station of an Aluminium plant, as the subsystem performance contribute to the entire functioning of the plant. Six years’ maintenance data on failures, repairs and various associated costs are collected for the purpose of this analysis. Three types of maintenance noted for the subsystem viz., corrective maintenance, inspection as proactive maintenance and service on requirement. Measures of subsystem effectiveness such as mean time to subsystem failure, availability of the system, busy period analysis and expected number of visit by the repairman for repair have been obtained. Semi-Markov processes and regenerative point techniques are used in this analysis.

Keywords - reliability, failure, repair, preventive maintenance, semi Markov process, regenerative point.

Notations and Symbols: 

\( f_i(t), F_i(t) \) of failure rate of the station \( i \)

\( g_i(t), G_i(t) \) of repair time of the station \( i \)

\( q_{ij}, Q_{ij} \) Probability density function (p.d.f) cumulative distribution function (c.d.f) from a regenerative state \( i \) to a regenerative state \( j \) without visiting any other regenerative state \( 0, t \).

\( P_{ij} \) Probability of transition from a regenerative state \( i \) to a regenerative state \( j \) in \( 0, t \).

\( S_i \) State \( i \).

\( \lambda_i \) Failure rate of \( i^{th} \) type

\( \alpha_i \) Repair rate of \( i^{th} \) type

\( \text{LTF} \) Symbol for a Laplace Transform

\( \text{USTLT} \) Symbol of a Laplace-Stieltjes transform

\( m_{ij} \) Unconditional mean time taken to transit to any regenerative state from the epoch of entry into regenerative state \( j \).

\( \mu_i \) Mean sojourn time in the regenerative state \( i \) before transitioning to any other state.

\( \Phi_i(t) \) Cumulative distribution function (c.d.f) of the first passage time from a regenerative state \( i \) to a failed state

\( M_i(t) \) The probability that the subsystem initially up in regenerative state \( i \), is up at a time \( t \) without going to any regenerative state

\( A_i(t) \) The probability of the unit entering into upstate at instant \( t \), giving that the unit entered in regenerative state \( i \) at \( t = 0 \)

\( B_i(t) \) Probability that the repairman is busy in inspection of instant \( t \), given that the system entered regenerative state \( i \) at \( t = 0 \)

\( V_i(t) \) Expected number of visits of the repairman, given that the subsystem entered regenerative state \( i \) at \( t = 0 \)

\( W_i(t) \) Probability that the repairman is busy in regenerative state \( i \) at time \( t \) without passing any other regenerative state.

I. INTRODUCTION

Complex industrial systems are subject to failures because of many reasons which affect the profitability of the industry and hence reliability analysis plays an important role in understanding the system performance while dealing with real industrial problems under different operating conditions and assumptions. Gulshan et al. [1] analyzed system with perfect repair under partial failure mode and priority for repair to completely failed unit, Gopalan & Bassi [2] considered two unit repairable system subject to on-line preventive maintenance and/or repair, Tuteja et al. [3]-[5] worked for two-units system with regular repairman who is not always available, system with perfect repair at partial failure or complete failure mode, and the profit evaluation of a two-units cold standby system with tiredness and two types of repairmen. Rizwan et al. [6]-[12] analyzed cold and hot standby systems with single-unit and two-units under different failure and repair situations and the some important reliability indices are obtained along with the cost benefit analysis of the systems. Mathew et al. [13]-[19] extensively analyzed the continuous casting plant and studied the variations under different operating conditions of the plant. Detailed analysis was reported for desalination plant by Padmavathi et al. [20] with online repair under emergency shutdowns, Rizwan et al. [21] with repair/maintenance strategy on first come first served basis, Padmavathi et al. [22]-[26] continued on desalination plant with priority for repair over maintenance, comparative analysis between the plant models, analysis under major and minor failures consideration, analysis by prioritizing repair over maintenance under major / minor failures, and comparative analysis between the plant models portraying two operating conditions of the plant as to which model is better than the other. The methodology was further extended for various industrial systems analyses by Gupta and Gupta [27] with post inspection concept, Ram et al. [28] waiting repair strategy, Malhotra and Taneja [29] both units operative on demand, Niwas et al. [30] obtained mean time to system failure and profit of a single unit system with inspection for feasibility of repair beyond warranty. Later, Rizwan et al. [31]-[33] focused on waste water treatment plant & anaerobic batch reactor and reliability indices of interest were obtained in order to assess the plant/reactor performance. Taj et al. [34] analyzed a single machine subsystem of a cable plant with six maintenance categories. Hence, the methodology is quite familiar for system analysis and has been widely presented in the literature, and proved to be a useful tool for system analysis.

Aluminum being widely used as a source input for manufacturing industries, therefore, is a good reason for this analysis from reliability perspective. One such aluminum manufacturing industrial plant operating in Oman has been considered for this purpose, and the analysis for a subsystem called butt shot blast station is carried out, as the subsystem performance contribute to the entire functioning of the plant. The plant manufactures raw aluminum blocks. Six years maintenance data on component failures, repairs and various associated costs are collected from the maintenance record. Three types of maintenance noted for the subsystem viz., corrective maintenance, inspection as proactive maintenance and service on requirement. Failure and repair rates with respect to maintenances are estimated from the data. Plant has eight stations viz., butt shot blast station 1 which is a subsystem of the plant, butt & thimble removal press station 2 with standby arrangement, combined btp (butt & thimble press) station 3, stub straighten station 4, stub shot blast station 5, stub coating and drying station 6, casting station 7, and anode rod inspection station 8. The plant operates round the clock, and failure in any of the stations impacts the plant to a complete shutdown situation. Reliability results at this level could be
useful measures in gauging and comparing the entire plant operational effectiveness. The state transitions of the subsystem are shown in Table 1. Semi-Markov process and regenerative point techniques are used in this analysis. Outcome of the subsystem analysis is measured in terms of mean time to system failure, availability of the subsystem, expected busy period of the repairman, and expected number of visits for repair.

A. DESCRIPTION OF THE SUBSYSTEM (BUTT SHOT BLAST STATION)

The subsystem transition states are:
State 0 (S₀): operative state
State 1 (S₁): failed state under repair or corrective maintenance.
State 2 (S₂): downstate under service on requirement.
State 3 (S₃): downstate under inspection as proactive maintenance.

The subsystem regenerates and works as good as new after every maintenance performed. Table 1 shows the transition rates from state S₀ to S₃. 0 denotes for no transition to the mentioned state. Failure rates are exponential whereas the repair rates are taken as general.

<table>
<thead>
<tr>
<th>S₀</th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₀</td>
<td>0</td>
<td>λ₁</td>
<td>λ₂</td>
</tr>
<tr>
<td>S₁</td>
<td>q₁(t)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₂</td>
<td>q₂(t)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S₃</td>
<td>q₃(t)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- All necessary maintenances are off-line which means plant need to be in switch-off mode.
- Maintenances need to be addressed on requirement by a single repairman.
- Other than failures which are exponentially distributed all distributions are general.

Table 2 shows the estimated values of repair/failure rates for the subsystem from the maintenance data of the plant.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Rate (per hour)</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>λ₁</td>
<td>0.01863041</td>
</tr>
<tr>
<td>2</td>
<td>λ₂</td>
<td>0.01637168</td>
</tr>
<tr>
<td>3</td>
<td>λ₃</td>
<td>0.00582265</td>
</tr>
</tbody>
</table>

B. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

State 0, 1, 2 and 3 are the regenerative states where 2 & 3 are the down states. The transition probabilities from Sᵢ to Sⱼ be given by the following equations:

\[ dQ_{01} = \lambda_1 e^{-(t_0 + t_2 + t_3)} dt \]  \hspace{1cm} (1)

\[ dQ_{02} = \lambda_2 e^{-(t_0 + t_2 + t_3)} dt \]  \hspace{1cm} (2)

\[ dQ_{03} = \lambda_3 e^{-(t_0 + t_2 + t_3)} dt \]  \hspace{1cm} (3)

\[ dQ_{10} = g_1(t) dt \]  \hspace{1cm} (4)

\[ dQ_{20} = g_2(t) dt \]  \hspace{1cm} (5)

\[ dQ_{30} = g_3(t) dt \]  \hspace{1cm} (6)

The non-zero elements \( p_{ij} = \lim_{s \to 0} q_{ij}'(s) \) are given below:

\[ p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \]  \hspace{1cm} (7)

\[ p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \]  \hspace{1cm} (8)

\[ p_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \]  \hspace{1cm} (9)

\[ p_{10} = 1 \]  \hspace{1cm} (10)

\[ p_{20} = 1 \]  \hspace{1cm} (11)

\[ p_{30} = 1 \]  \hspace{1cm} (12)

By these transition probabilities it can be verified that:

\[ p_{01} + p_{02} + p_{03} = 1 \]  \hspace{1cm} (13)

\[ p_{10} = p_{20} = p_{30} = 1 \]  \hspace{1cm} (14)

The mean sojourn time \( \phi_i \) in regenerative state i is defined as the time of stay in that state before transition to any other state. So, if \( T \) denotes the sojourn time in the regenerative state i then:

\[ \mu_i = \int_0^\infty t dQ_i(t) \]  \hspace{1cm} (15)

\[ \mu_0 = \int_0^\infty t dQ_{01}(t) + \int_0^\infty t dQ_{02}(t) + \int_0^\infty t dQ_{03}(t) \]  \hspace{1cm} (16)

\[ \mu_1 = \int_0^\infty t g_1(t) dt \]  \hspace{1cm} (17)

\[ \mu_2 = \int_0^\infty t g_2(t) dt \]  \hspace{1cm} (18)

\[ \mu_3 = \int_0^\infty t g_3(t) dt \]  \hspace{1cm} (19)

The unconditional mean time taken by the system to transit for any state \( j \) when it has taken from epoch of entrance into regenerative state \( i \) is mathematically stated as:

\[ m_{ij} = \lim_{s \to 0} \frac{d}{ds} [q_{ij}'(s)] \]  \hspace{1cm} (20)

Thus,

\[ m_{01} + m_{02} + m_{03} = \mu_0 \]  \hspace{1cm} (21)

\[ m_{10} = \mu_1 \]  \hspace{1cm} (22)

\[ m_{20} = \mu_2 \]  \hspace{1cm} (23)

\[ m_{30} = \mu_3 \]  \hspace{1cm} (24)

II. MATHEMATICAL ANALYSIS

A. MEAN TIME TO SYSTEM FAILURE

Let \( \phi_i(t) \) be the c.d.f of the first passage time from regenerative state \( i \) to a failed state. By probabilistic arguments, the following recursive relation for \( \phi_i(t) \) are obtained:

\[ \phi_0(t) = Q_{01}(t) + Q_{02}(t) + Q_{03}(t) \]  \hspace{1cm} (25)

On taking Laplace Stieltjes transform of equation (25) and solving for \( \phi_0''(s) \), the mean time to system failure in steady state is given by:

\[ \text{MTSF} = \lim_{s \to 0} \frac{1}{s} \phi_0''(s) = \frac{N}{D} \]  \hspace{1cm} (27)

Where, \( N = \mu_0 \) and \( D = 1 \).
B. AVAILABILITY ANALYSIS OF THE SYSTEM

\( A_1(t) \) is the probability of the unit entering into the upstate at an instant \( t \), given that the unit entered in regenerative state \( i \) at \( t = 0 \). The following recursive relations are obtained for \( A_i(t) \):

\[
A_2(t) = M_0(t) + Q_{02}(t) A_1(t) + Q_{03}(t) A_2(t)
\]

\[ + Q_{05}(t) A_3(t) \]  

\( A_1(t) = Q_{01}(t) A_0(t) \)  

\( A_3(t) = Q_{02}(t) A_0(t) \)  

\( A_4(t) = Q_{03}(t) A_0(t) \)  

Where \( M_0(t) = e^{-t(A_1 + A_2 + A_3)} \)

On taking Laplace transforms of the equations (28) to (31) and solving them for \( A_i(s) \), the availability of the subsystem in steady state is given by:

\[
A_s = \lim_{s \to 0} A_i(s) = \frac{N_i}{D_i}
\]  

Where, \( N_i = \mu_0 + D_i = \mu_0 + u_1 \mu_1 + \mu_2 \mu_2 + \mu_3 \mu_3 \)

C. BUSY PERIOD ANALYSIS OF REPAIRMAN

Using the probabilistic arguments, we have the following relations for \( B_i(t) \) as probability that the repairman is busy for repair at instant \( t \), given that unit entered in regenerative state \( i \) at \( t = 0 \), the following recursive relations are obtained for \( B_i(t) \):

\[
B_0(t) = Q_{02}(t) B_1(t) + Q_{03}(t) B_2(t) + Q_{05}(t) B_3(t)
\]  

\( B_1(t) = W_1(t) + Q_{10}(t) B_0(t) \)  

\( B_2(t) = W_2(t) + Q_{20}(t) B_1(t) \)  

\( B_3(t) = W_3(t) + Q_{30}(t) B_2(t) \)  

Where, \( W_i(t) = g_i(t); W_2(t) = g_2(t); W_3(t) = g_3(t) \)

On taking the Laplace transforms of the equations (33) to (36), the expected busy period of the repairman in steady state is given by:

\[
B_s = \lim_{s \to 0} B_i(s) = \frac{N_i}{D_i}
\]  

Where, \( N_i = \mu_0 + D_i = \mu_0 + u_1 \mu_1 + \mu_2 \mu_2 + \mu_3 \mu_3 \) and \( D_i \) is already specified

D. EXPECTED NUMBER OF VISITS BY THE REPAIRMAN FOR REPAIRS

Let \( V_i(t) \) be defined as the expected number of visits for repairs in \([0, t]\) given that the system initially starts from the regenerative state \( i \). Using the probabilistic arguments, the following recursive relations are obtained for \( V_i(t) \):

\[
V_0(t) = Q_{02}(t) V_1(t) + Q_{03}(t) V_2(t)
\]

\[ + Q_{05}(t) (1 + V_0(t)) \]  

\( V_1(t) = Q_{01}(t) V_0(t) \)  

\( V_2(t) = Q_{02}(t) V_1(t) \)  

\( V_3(t) = Q_{03}(t) V_2(t) \)  

Taking Laplace Stieltje's transform of the above equations, number of visits by the repairman in steady state is given by:

\[
V_s = \lim_{s \to 0} sV_i^\prime(s) = \frac{N_i}{D_i} \frac{N_s}{V_i}
\]

Where,

\( N_s = \mu_0 + \mu_1 + \mu_2 \mu_2 + \mu_3 \mu_3 + D_i \) (Already specified)

III. PARTICULAR CASE

For this particular case, the following have been considered:

\[
g_1(t)dt = a_1 e^{-a_1 t}; g_2(t)dt = a_2 e^{-a_2 t}; g_3(t)dt = a_3 e^{-a_3 t}
\]

Where, \( a_1 = 0.19080361; a_2 = 0.39290235 \) and \( a_3 = 0.13974359 \)

Using the data as summarized in table 2, the expressions of reliability measures as in (27), (32), (37), and (42), the following values of subsystem effectiveness are obtained:

- Mean time to system failure = 58.1487 hrs.
- Availability = 0.846758 hrs.
- Busy period of repairman = 0.153242
- Expected number visits by the repairman for repair = 0.158173

IV. CONCLUSION

Mean time to system failure is about 58 hours which shows, there is a failure almost every 58 hours. Other measures could further be improved by adopting better maintenance practices. As a future direction, the analysis could further be explored for the entire plant.

REFERENCES